Lytchett Minster School

Year 9
Mathematics
Knowledge organisers

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2024/2025

Proportion

Keywords and Phrases:

Proportion - a part, a share, or number considered in comparative relation to a whole.

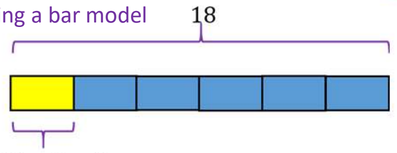
Percent – The parts per 100, a ratio “out of 100”

Fractions of an amount:

Example 1: Find $\frac{1}{6}$ of 18

$\frac{1}{6}$ 1 of the equal parts; 6 equal parts

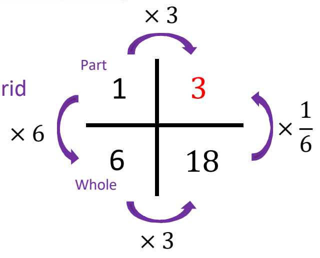
Using a bar model



$18 \div 6 = 3$

$18 \times \frac{1}{6} = 3$

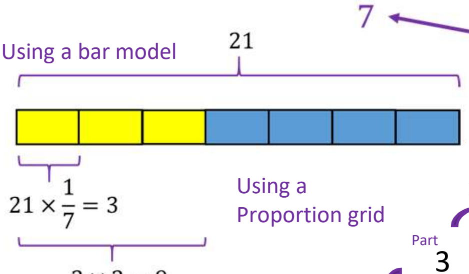
Using a Proportion grid



Example 2: Find $\frac{3}{7}$ of 21

$\frac{3}{7}$ Multiply by 3; Divide by 7

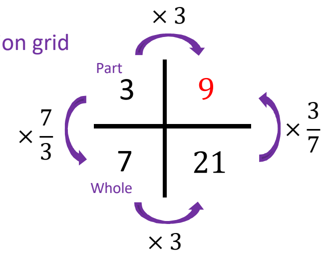
Using a bar model



$21 \times \frac{1}{7} = 3$

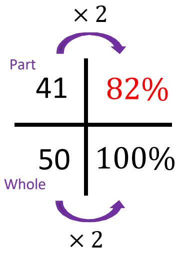
$3 \times 3 = 9$

Using a Proportion grid



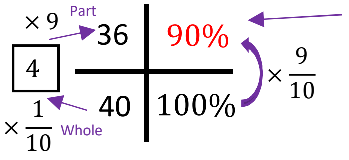
Writing a Number as a Percentage of a Quantity – Non Calc

Example 1: Bob scored 41 out of 50 on his test.
What is this as a percentage?



$\frac{41}{50} = 82\%$

Example 2: Jane scored 36 out of 40 on her test.
What is this as a percentage?

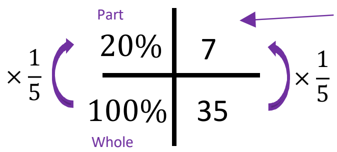


$100 \times \frac{9}{10} = 90$

$\frac{36}{40} = 90\%$

Percentages of an amount

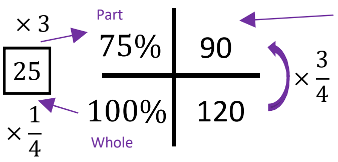
Example 1: Find 20% of 35



$35 \times \frac{1}{5} = 7$

20% of 35 = 7

Example 2: Find 75% of 120



$120 \times \frac{3}{4} = 90$

75% of 120 = 90

Your turn to practice

1) Find $\frac{2}{3}$ of 9.

2) Find $\frac{3}{5}$ of 15.

3) Find $\frac{3}{4}$ of 8.

4) Find $\frac{6}{11}$ of 55.

5) Find $\frac{4}{5}$ of 20.

6) 9 out of 25

7) 34 out of 50

8) 14 out of 20

9) 17 out of 25

10) 21 out of 30

11) Find 25% of 40

12) Find 25% of 80

13) Find 75% of 20

14) Find 75% of 200

15) Find 5% of 40

16) Find 5% of 60

17) Find 15% of 60

18) Find 35% of 65

19) Find 65% of 85

20) Find 75% of 160

Answers

1)	6	6)	76%	11)	10	16)	3
2)	9	7)	68%	12)	20	17)	9
3)	6	8)	70%	13)	15	18)	22.75
4)	30	9)	68%	14)	150	19)	55.25
5)	16	10)	70%	15)	2	20)	120

Proportion - Percentages

Keywords and Phrases:

Proportion - a part, a share, or number considered in comparative relation to a whole.

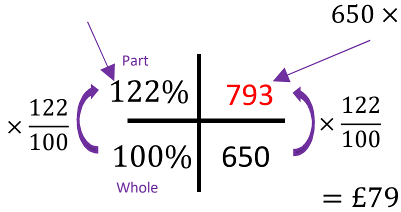
Percent – The parts per 100, a ratio “out of 100”

Increasing/Decreasing by a Percentage

An antique necklace cost £650. Its value is increased by 22%. What is it worth now?

$100\% + 22\% = 122\%$

$650 \times \frac{122}{100} = 793$

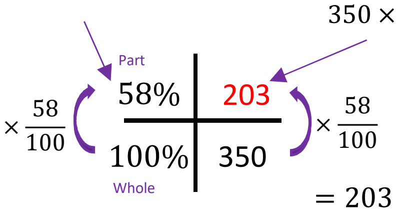


$= £793$

The number of number of students in the sixth form decreased of 42%. Last year we had 350 students how many are there now?

$100\% - 42\% = 58\%$

$350 \times \frac{58}{100} = 203$



$= 203$ students

Decimal multipliers

Decimal multipliers are very helpful to calculate an percentage.

12% of an amount $= \frac{12}{100} \times$ an amount $= 0.12 \times$ an amount

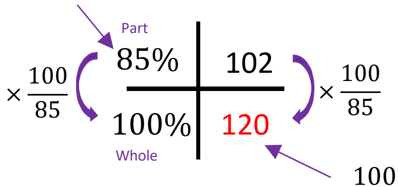
$12 \div 100 = 0.12$

This is the decimal multiplier we can use to find 12% of an amount

Reverse Percentages

A jacket costs £102 after a discount of 15%. What was the original price?

$100\% - 15\% = 85\%$

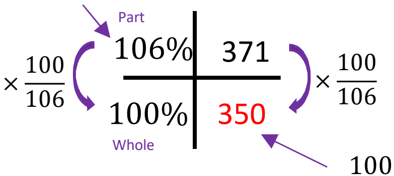


$102 \times \frac{100}{85} = 120$

$= £120$

The share price of a company increased by 6%. The price per share is 371p. Work out the share price before the increase?

$100\% + 6\% = 106\%$



$371 \times \frac{100}{106} = 350$

$= 350p$

Your turn to practice

1) Decrease 800 by 20%
2) Decrease 600 by 50%
3) Increase 40 by 10%
4) Increase 20 by 25%
5) Increase 30 by 40%
6) Decrease 800 by 95%
7) Decrease 600 by 75%
8) Increase 40 by 15%
9) Increase 120 by 35%
10) Increase 180 by 65%

Find the original whole given the following information:

11) A 30% increase gives 260.
12) A 40% increase gives 70.
13) A 25% increase gives 35.
14) A 10% decrease gives 180.
15) A 70% decrease gives 420.

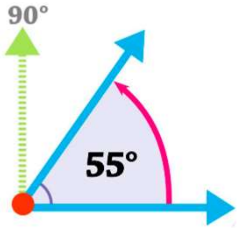
Answers

1)	640	6)	40	11)	200
2)	300	7)	150	12)	50
3)	44	8)	46	13)	28
4)	25	9)	162	14)	200
5)	42	10)	297	15)	1400

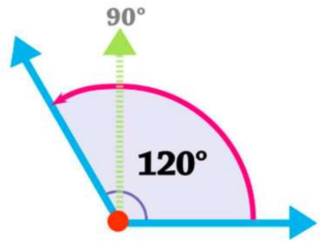
Angles

Keywords and Phrases:

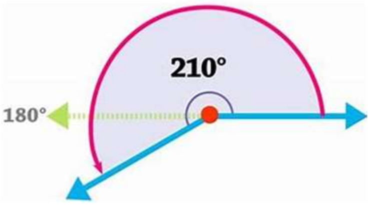
Acute – An angle less than 90°



Obtuse – An angle bigger than 90° but less than 180°



Reflex – An angle bigger than 180°



Equipment used to measure or draw an angle is a protractor

1. Place the small hole or vertical line at the bottom centre of your protractor over the vertex of your angle, or where the 2 lines meet.
2. Adjust your protractor so 1 of the angle's rays, or lines, is straight and flush against the 0° measurement.
3. Start at 0° and follow the scale measurements up to where the other line passes through the protractor.
4. Find the cross or circle in the middle of your protractor.
5. Place this point at the vertex of the angle you are measuring.
6. Read from zero on the outer scale of your protractor.
7. Count the degree lines carefully.
8. Record your final measurement in degrees (°)

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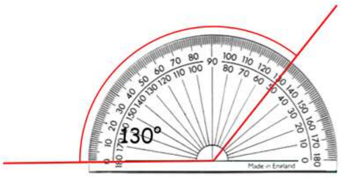
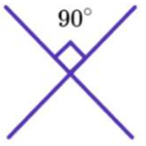


Key phrases and notation

A line that is between two points is known as a line segment



Perpendicular Lines

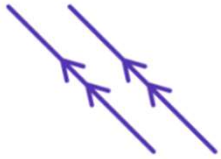
Perpendicular lines always intersect (cross) at a right angle. The gradients are different

⊥ means perpendicular

∥ means "is parallel to"

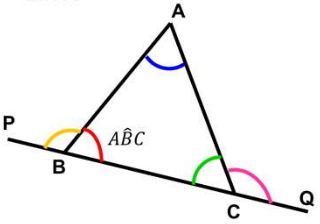
AC ∥ DF

AC ⊥ BK



Parallel Lines

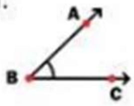
Parallel lines never meet and the arrows indicate that the they are parallel in diagrams. The gradient of the line is the same



An angle is created by two line segments AB and BC. We note that B is the common point known as the vertex. We use the notation **$A\hat{B}C$. The** ⌒ is the common point

Your turn to Practise

For each of the following angles (i) Name the angle (ii) Label the angle with the correct notation (iii) Measure the angle with a protractor

1.


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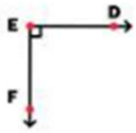

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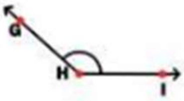

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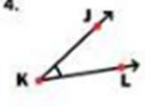
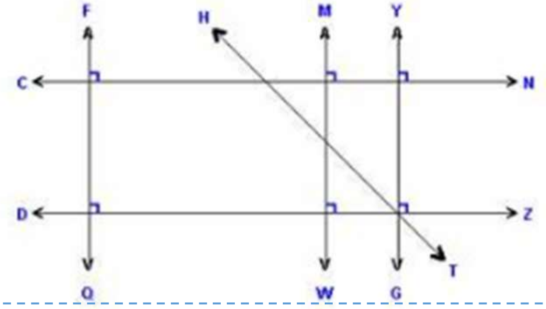


5 Identify the parallel, perpendicular and intersecting lines



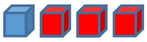
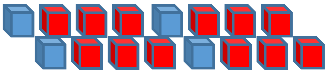
Answers: 1) Acute $A\hat{B}C$ ...., 2) Right angle $D\hat{E}F$ ...., 3) Obtuse $G\hat{H}I$ ......, 4) Acute $J\hat{K}L$ ..... 5) FQ ∥MW ∥YG, CN ∥DZ, CN and DZ ⊥ FQ MW and YG

Ratio and Equivalent Ratio

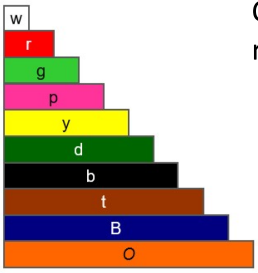
Keywords and Phrases:

Ratio and Proportion describe the multiplicative relationship between two numbers.

The order the words are written in a question correspond to the order of the numbers in the ratio.
E.g. The ratio of blue cubes to red cubes is 1:3.
This means for every blue cube there are three red cubes (three times as many red cubes as blue)

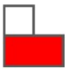
E.g.  or 

Examples and key points

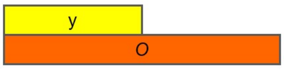


Cuisenaire rods can be used to help us visualise the multiplicative relationship between numbers

E.g. The length of the second rod is twice the length of the first rod



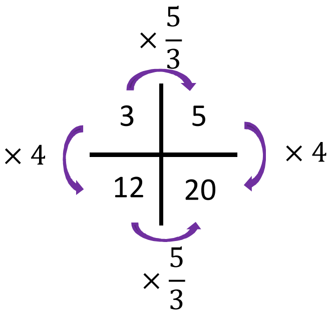
$w : r$
$1 : 2$



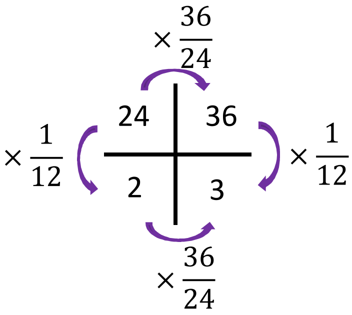
y: O
$1 : 2$

It will help if you can remember how to find the highest common factor of two numbers
E.g.
The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24
The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36
The common factors of 24 and 36 are 1, 2, 3, 4, 6, 12
The highest common factor of 24 and 36 is 12

We can use a proportion grid to find equivalent ratio, e.g. 3 : 5 is equivalent to 12 : 20



We can use a proportion grid to simplify a ratio, e.g. 24 : 36 simplifies to 2 : 3



Your turn to practice

Write these ratios in their simplest forms

(1) 2 : 6
(2) 9 : 3
(3) 10 : 15
(4) 4 : 12
(5) 14 : 21
(6) 36 : 48
(7) 45 : 18
(8) 42 : 56
(9) 64 : 48
(10) 90 : 75
(11) 2⅕ : 1⅓ : 1⅔
(12) 1¼ : 2
(13) 9 : 3⅜ : ⅖
(14) 1¼ : 1⅔
(15) 2⅝ : 1¾

Answers
1) 1 : 3
2) 3 : 1
3) 2 : 3
4) 1 : 3
5) 2 : 3
6) 3 : 4
7) 5 : 2
8) 3 : 4
9) 4 : 3
10) 6 : 5
11) 33 : 20 : 25
12) 5 : 8
13) 360 : 135 : 16
14) 15 : 20 = 3 : 4
15) 21 : 14 = 3 : 2

Angles

Keywords and Phrases:

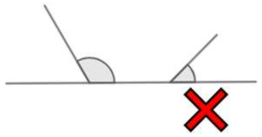
"Vertically opposite" in this topic means that the opposite angles meet at a vertex

A **Vertex** is a point

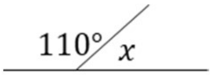
Adjacent means angles that are next to each other AND touching

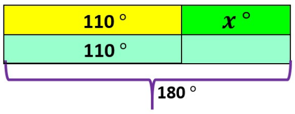
Examples and key points

Adjacent Angles on a straight line sum to 180°

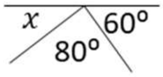


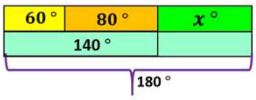
These angles are NOT adjacent (touching) so the rule is not true!





$$110 + x = 180$$
$$\overset{0}{\cancel{110}} + x = \overset{0}{\cancel{110}} + 70$$
$$-\cancel{110} \qquad -\cancel{110}$$
$$x = 70°$$

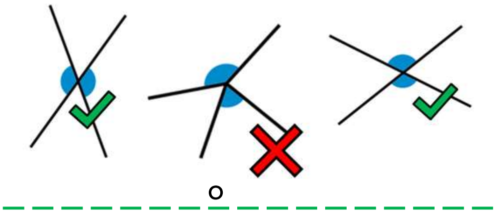


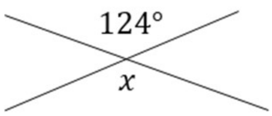


$$x + 80 + 60 = 180$$
$$x + 140 = 180$$
$$x + \overset{0}{\cancel{140}} = 40 + \overset{0}{\cancel{140}}$$
$$-\cancel{140} \qquad -\cancel{140}$$
$$x = 40°$$

Vertically opposite angles are equal

"Vertically" opposite means angles that are opposite one another and touch at a point. This is only true when the diagram shows two straight lines intersecting!

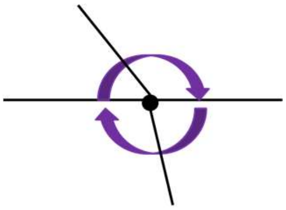




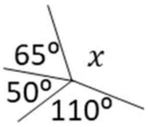
$x = 124°$

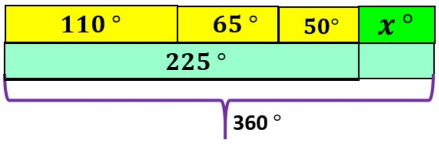
Angles that meet at a point sum to 360°

As we know, adjacent angles on a straight line sum to 180°,
So as we can see in the diagram below, angles that meet at (around) a point will add up to 180°+180°=360°



$180° + 180° = 360°$

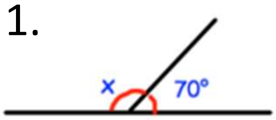




$$110 + 65 + 50 + x = 360$$
$$225 + x = 360$$
$$\overset{0}{\cancel{225}} + x = \overset{0}{\cancel{225}} + 135$$
$$-\cancel{225} \qquad -\cancel{225}$$
$$x = 135°$$

Your turn to practise

1.



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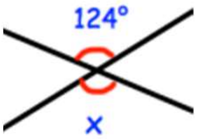

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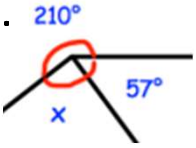

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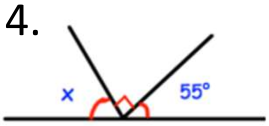

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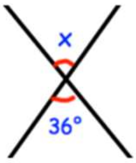

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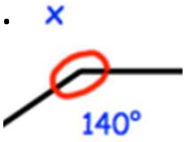

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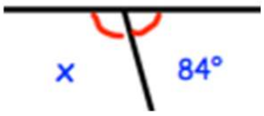

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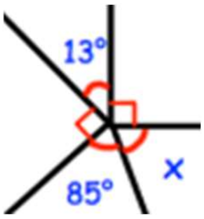

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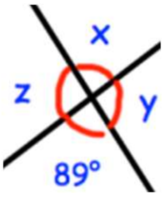


ANSWERS
1. x=110°
2. x=56°
3. x=93°
4. x=35°
5. x=36°
6. x=220°
7. x=96°
8. x=82°
9. x=89°, y=91°, z=91°

Angles in Parallel Lines

Keywords and Phrases:

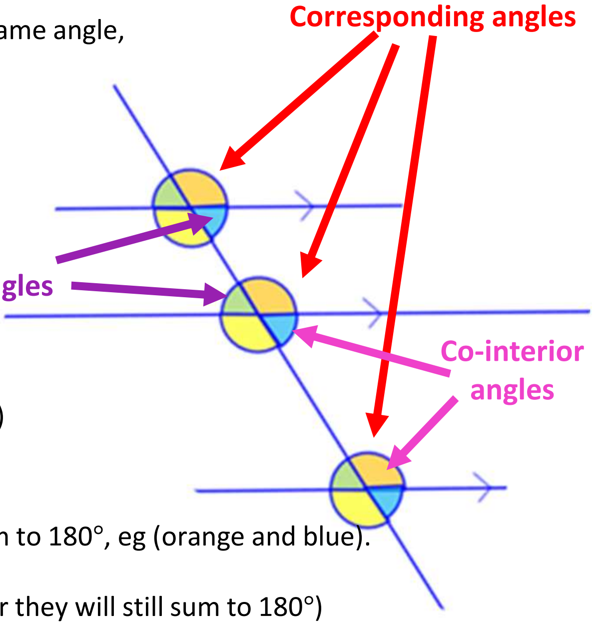
Parallel – lines that are travelling at the same angle (and will never meet)

Transversal – a line that intersects with a pair or group of parallel lines

“corresponding” in this topic means that the angles are in the same position in relation to the parallel lines and the transversal (see Angles 1)

Examples and key points

Because the parallel lines all intersect the transversal at the same angle, we can see that groups of equal angles are formed.



Corresponding angles are equal

Corresponding angles have the same position relative to the parallel lines and transversal.
Eg the green angles are all ABOVE the parallel lines and LEFT of the transversal. Therefore they are all equal.

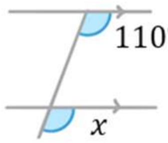
Alternate angles are equal

Alternate angles are pairs of angles INSIDE the parallel lines, but on ALTERNATE sides of the transversal
(eg. one is on the left (blue), the other is on the right (green))

Co-interior angles sum to 180°

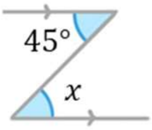
As we know (Angles 2a), adjacent angles on a straight line sum to 180°, eg (orange and blue).
These also appear INSIDE the parallel lines but apart.
We call these co-interior angles (and as they are the same pair they will still sum to 180°)

Corresponding angles



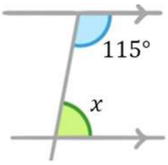
$x = 110°$

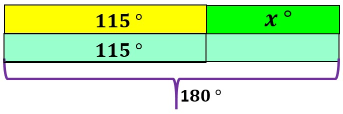
Alternate angles



$x = 45°$

Co-interior angles





$$115 + x = 180$$

$$115 + x = 115 + 65$$

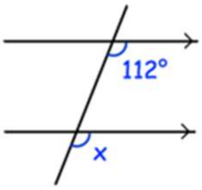
$$-115 \qquad -115$$

0 0

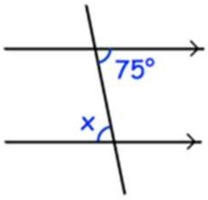
$$\boxed{x = 65°}$$

Your turn to practise

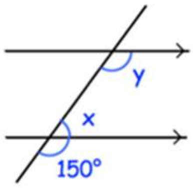
(a)



(b)



(c)



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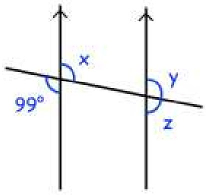
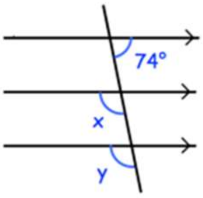
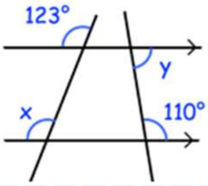


(e)



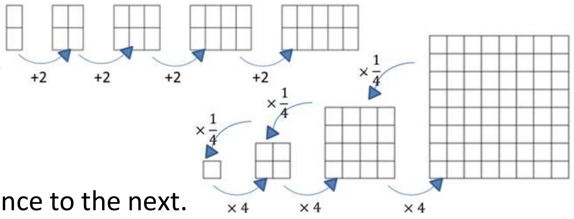
(f)



Answers
a) x=112° (corresponding)
b) x=75° (alternate)
c) x=30°(straight line),
y=150°(corresponding)
d) x=99° (vertically opposite)
y=99° (corresponding to x)
z=81° (straight line with y)
e) x=106°, y=106°
f) x=123° (corresponding)
y=70° (co-interior)

Sequences

Keywords and Phrases:



Arithmetic Sequences – Change by a common difference. This is found by adding or subtracting between terms.

Geometric Sequences – Change by a common ratio This is found by multiplication / division between terms:

Term to term rule – How to get from one term in a sequence to the next.

Position to term rule – Represented by the function machine that says how the number in the sequence from the position.

Recurrence relation - A recurrence relation is a rule that gives you a connection between two consecutive terms. This connection can be used to find next/previous terms.

U_1	U_2	U_3	U_4	U_5	… U_n
1	5	9	13	17	… ?

+4 +4 +4 +4

$$\ldots\ldots U_{n+1} = U_n + 4$$ ← Recurrence relation

nth term - The nth term refers to the position of a term in a sequence (position to term rule). For example the first term has n=1, the second term has n=2, the 10th term has n=10 and so on. The 'n' stands for its position number in the sequence.

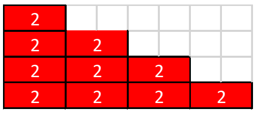
Arithmetic sequences – n^{th} term

All our times tables are arithmetic sequences because the terms have a common difference. For example the three times table has a common difference of 3.

An arithmetic sequence is a times table that has shifted by a given value, you can see this visually using rods.

Two times table:

2, 4, 6, 8, …

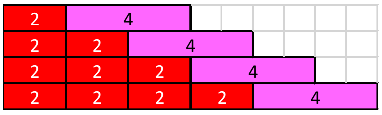


The common difference, d = 2

The shift is 0

$nth\ term = 2n$

6, 8, 10, 12, …

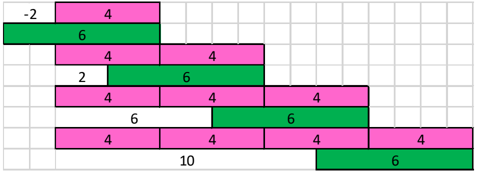


The common difference, d = 2

The shift is +4

$nth\ term = 2n + 4$

-2, 2, 6, 10, …



The common difference, d = 4

The shift is −6

$nth\ term = 4n - 6$

Other sequences

Fibonacci

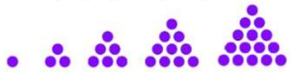
Each term is a sum of the previous two terms

1, 1, 2, 3, 5, 8, 13, ….

Triangular numbers

The formation of the numbers create triangles

1, 3, 6, 10, 15, ….



Square numbers

The formation of the numbers create squares

1, 4, 9, 16, 25, ….

Your turn to practise

Find the nth term for each of the following sequences

1) 5, 8, 11, 14, … …
2) 9, 14, 19, 24, … …
3) 1, 3, 5, 7, … …
4) 10, 14, 18, 22, … …
5) 2, 7, 12, 17, … …
6) 10, 7, 4, 1, … …
7) 6, 4, 2, 0, … …
8) 9, 4, -1, -6, … …
9) 20, 10, 0, -10, … …
10) 5, -1, -7, -13, … …

The nth term for some sequences are given below. Find the first 5 terms for each sequence.

11) 5n + 3
12) 2n + 9
13) 3n – 2
14) 10n – 6
15) 9n + 10

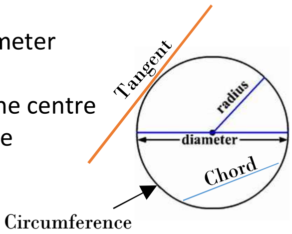
Answers

1) 3n+2
2) 5n+4
3) 2n-1
4) 4n+6
5) 5n-3
6) -3n+13
7) -2n+8
8) -5n+14
9) -10n+30
10) -6n+11
11) 8,13,18,23,28
12) 11,13,15,17,19
13) 1,4,7,10,13
14) 4,14,24,34,44
15) 19,28,37,46,55

Circles

Keywords and Phrases:

Circumference: the length around the outside of the circle – the perimeter
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
The radius is half the diameter.
Tangent: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve



Pi ($\boldsymbol{\pi}$) Is the ratio between the **circumference** of a circle and its **diameter**. This ratio is given a Greek letter and is always equal to **3.14159265358…..**
Pi (π) is an irrational number and can never be recorded accurately using digits without rounding. Knowing Pi to 39 digits is enough to work out the circumference of the universe to within the width of a hydrogen atom.

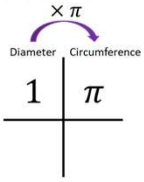
Key Facts – Circle Formulae

Area of a circle
$A = \pi r^2$

The area of a circle will always give an answer measured in un ^{2}
This formula can also be used as the basis for finding the area of sectors and volume of cylinders.

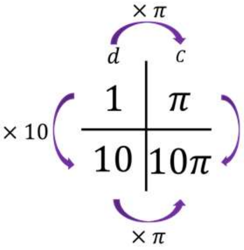
Circumference of a circle
$C = \pi d$
$C = 2\pi r$

The circumference of a circle will always give an answer measured in *units*. This formula can also be used to calculate the perimeter of shapes made up from parts of a circle.



Circumference of a circle

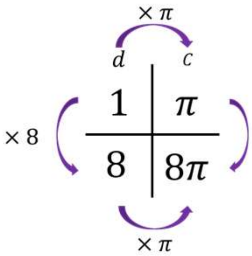
Find the circumference of a circle with a diameter of 10 cm first in terms of π



$C = 10\pi$ cm

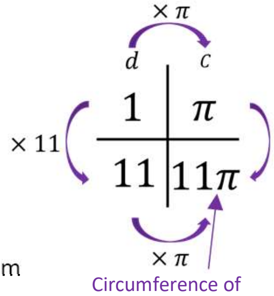
Find the circumference of a circle with a **radius** of 4 cm in terms of π

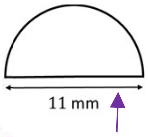
Careful we need diameter



$C = 8\pi$ cm

Find the **perimeter** of this shape leaving your answer in terms or π





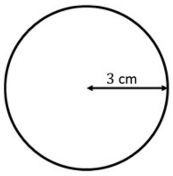
For perimeter add on diameter

Circumference of whole circle, so ÷ 2

$$P = \frac{11}{2}\pi + 11$$

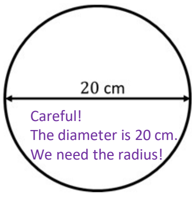
Area of a circle

Find the area of the circle, leaving your answer in terms of π



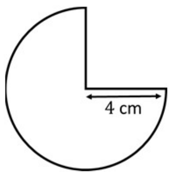
$\text{Area} = \pi r^2$
$= \pi \times r^2$
$= \pi \times 3^2$
$= \pi \times 9$
$= 9\pi$ cm^2

Find the area of the circle, leaving your answer in terms of π



$\text{Area} = \pi r^2$
$= \pi \times r^2$
$= \pi \times 10^2$
$= \pi \times 100$
$= 100\pi$ cm^2

Find the area of the shape, leaving your answer in terms of π

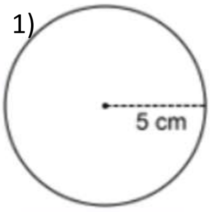


$\text{Area} = \frac{3}{4}\pi r^2$
$= \frac{3}{4} \times \pi \times 4^2$
$= \frac{3}{4} \times \pi \times 4 \times 4$
$= \pi \times \frac{4}{4} \times 3 \times 4$
$= 12\pi$ cm^2

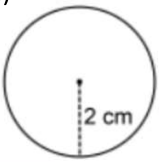
Your turn to practice

Calculate the area and circumference of the following, leaving your answer in terms of π

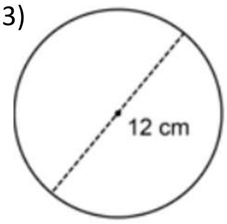
1)



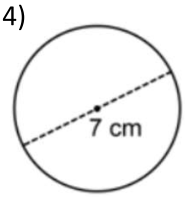
2)



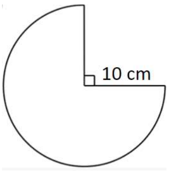
3)



4)



5)

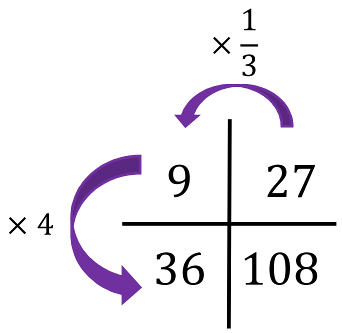


Answers
1) $A = 25\pi,\ C = 10\pi$
2) $A = 4\pi,\ C = 8\pi$
3) $A = 36\pi,\ C = 12\pi$
4) $A = 12.25\pi,\ C = 7\pi$
5) $A = 7\pi,\ C = 15\pi$

Recipes

Keywords and Phrases:

Recipes problems use proportional reasoning



Examples and key points

Recipe 1 - Carrot Cake (serves 4 people)

450ml of vegetable oil

400g plain flour

2 tsp of bicarbonate of soda

5 eggs

½ tsp of salt

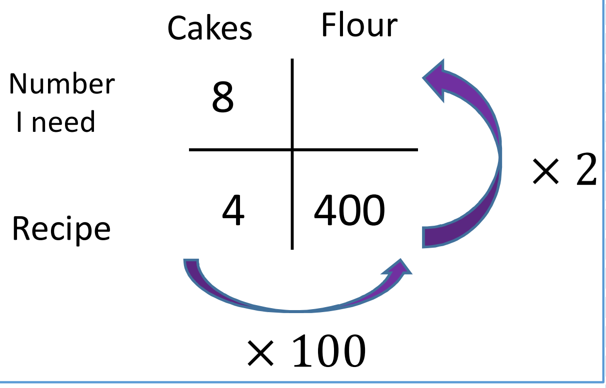
2½ tsp of ground cinnamon

525g of carrots

150g of chopped walnuts

I want to make this carrot cake for 8 people.
I have 1kg of flour.
Will I be able to make the cake?

Step 1 - Set up a proportion grid. Label the things you can match from the recipe and what you need



Step 2 – Clearly state your calculation

$8 \times 100 = 800 grams\ of\ flour$

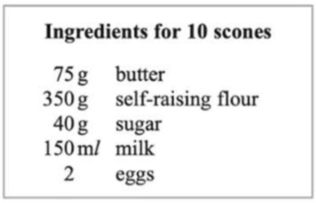
Or $400 \times 2 = 800 grams\ of\ flour$

Step 3 – Answer the question

1kg = 1000g
You need 800grams you have enough flour

Your turn to practice – Green is the easiest and red is the hardest

Here is a list of ingredients for making 10 scones

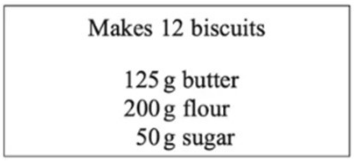


Ingredients for 10 scones

75 g	butter
350 g	self-raising flour
40 g	sugar
150 m*l*	milk
2	eggs

Mia wants to make 25 scones
Work out how much sugar she would need

Heidi wants to make some biscuits using the recipe



Makes 12 biscuits

125 g butter
200 g flour
50 g sugar

Heidi thinks that she has,
500g of butter
700g of flour
250g of sugar
What is the greatest number of biscuits Heidi can make?

Deon needs 50g of sugar to make 15 biscuits

She also needs
Three times as much flour as sugar
Two times as much butter as sugar

Deon is going to make 60 biscuits
Work out how much flour she needs

Answers: Q1 = 100 Q2 = 42 Q3 = 600

Pie Charts

Keywords and Phrases:

Ratio and Proportion describe the multiplicative relationship between two numbers.

Pie Charts are a pictorial representation of proportionality

To draw a pie chart you will need to use a protractor to measure and draw the angles. The QR code will help with this knowledge organiser as it is a lesson all about pie charts.

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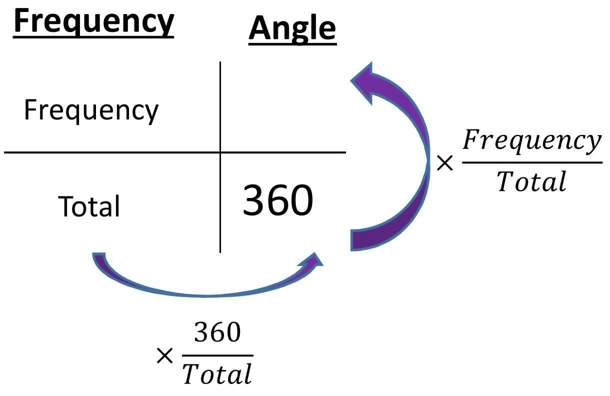
Key Features of a pie chart.

Pie charts are used to represent categorical data for example: colour of cars, different sports. Each sector is a proportion of the total so comparing them is easy

Set up a proportion grid. To find angles and frequencies in a pie chart



Frequency	**Angle**
Frequency	
Total	360

$\times \frac{Frequency}{Total}$

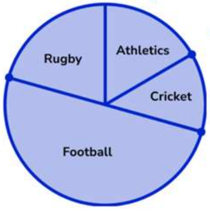
$\times \frac{360}{Total}$

Instructions

1. Calculate the angle for each category using proportion
2. Draw a circle, mar the centre and the radius
3. Measure and draw the angle for the first category
4. Measure and draw the angle for each category, in order
5. Add data labels/ complete a key

Sport	Frequency	Angle
Athletics	3 $\times 20$	$= 60$
Cricket	2 $\times 20$	$= 40$
Football	9 $\times 20$	$= 180$
Rugby	4 $\times 20$	$= 80$
Total	18	360

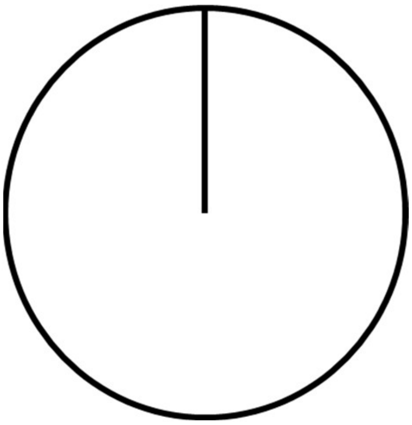
$\times \frac{360}{18}$



Your Turn to Practise:

Draw a pie chart for the following data using the circle

Car colour	Frequency
Blue	4
Green	15
Red	5
Yellow	12



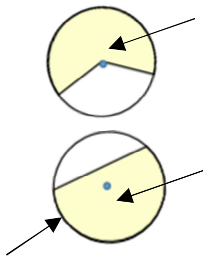
Answers: Blue: 40°, Green:150°, Red:50°, Yellow:120°

Arcs and Sectors

Keywords and Phrases:

Sector - A Sector is a section of a circle that is created by 2 radii and an arc.
Arc - An Arc is a portion of the circumference of a circle.

To calculate the area of a sector or an arc length you first calculate the area of the whole circle or the whole circumference. You can then use a proportion grid to calculate the sector area or arc length accordingly.



Sector (part of the circle made from two radii)

Segment (part of the circle made from a chord)

An arc is a part of the circumference

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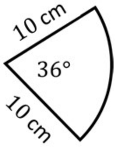
Area of a sector

1) Find the area of this sector
Leave your answer in terms of π:



$\times \frac{1}{10}$

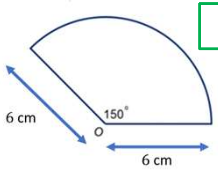
Sector 36°	10π
Full Circle 360°	100π

$\times \frac{1}{10}$

If this were a full circle…
Area = $\pi \times 10^2 = 100\pi$

The area of the sector
$= 100\pi$ cm²

2) Find the area of this sector
Leave your answer in terms of π:



If this were a full circle…
Area = $\pi \times 6^2 = 36\pi$

Highest Common Factor

$\times 5$ $\quad$ $\times 5$

Sector 150°	15π
Full Circle 360°	36π

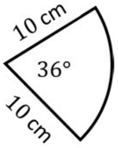
$\boxed{30}$ $\quad$ $\boxed{3\pi}$

$\times \frac{1}{12}$ $\quad$ $\times \frac{1}{12}$

The area of the sector
$= 15\pi$ cm²

Length of arc

1) Find the **perimeter** of the sector.
Leave your answer in terms of π:



$\times \frac{1}{10}$

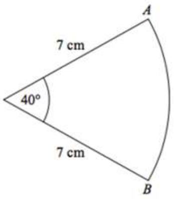
Arc 36°	2π
Full Circle 360°	20π

$\times \frac{1}{10}$

If this were a full circle…
Circumference = $\pi \times 20$
$= 20\pi$

The perimeter of the sector is
$2\pi + 10 + 10 = 2\pi + 20$ cm

2) Find the length of the arc AB:



$\times \frac{1}{9}$

Arc 40°	$\frac{14}{9}\pi$
Full Circle 360°	14π

$\times \frac{1}{9}$

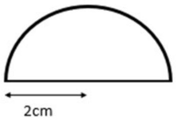
If this were a full circle…
Circumference = $\pi \times 14$
$= 14\pi$

The length of the arc is $\frac{14}{9}\pi$ cm

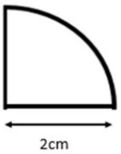
Your turn to practice

Calculate the area of the sector and the arc length of the following leaving your answer in terms of π

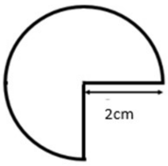
1)



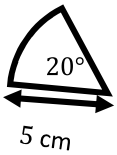
2)



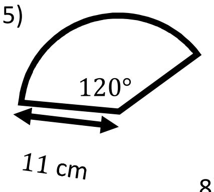
3)



4)

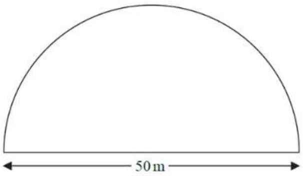


5)



6)

7) A farmer has a field in the shape of a semicircle of diameter 50 m.



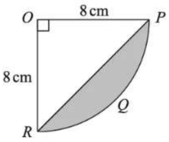
The farmer asks Jim to build a fence around the edge of the field.
Jim tells him how much it will cost.

Total cost = £29.86 per metre of fence plus £180 for each day's work

Jim takes three days to build the fence.

Work out the total cost.

8) The diagram shows a sector *OPQR* of a circle, centre *O* and radius 8 cm.



OPR is a triangle.

Work out the area of the shaded segment *PQR*.
Give your answer correct to 3 significant figures.

Answers

1) $A = 2\pi, Arc\ lengt\quad 2\pi$
2) $A = \pi, Arc\ length = \pi$
3) $A = 3\pi, Arc\ length = 3\pi$
4) $A = \frac{25}{18}\pi, Arc\ length = \frac{5}{9}\pi$
5) $A = \frac{121}{3}\pi, Arc\ length = \frac{22}{3}\pi$
6) $A = \frac{100}{9}\pi, Arc\ length = \frac{20}{9}\pi$
7) Perimeter = 128.54m
$129m \ \times £29.86 + (3 \times 180)$
$= £4378.20$
8) $A = 16\pi - 32 = 18.3cm^2\ (3s.f)$

Similarity

Keywords and Phrases:

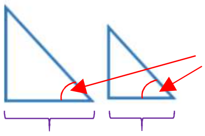
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.

Congruent: the same size and shape, but NOT an enlargement.

These shapes are congruent

These shapes are similar

Identify similar shapes

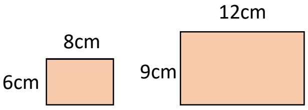


Angles in similar shapes do not change. e.g. if a triangle gets bigger the angles can not go above 180^0

These two sides are corresponding, we can compare all corresponding sides to find out if the same is similar.

Example

Area these shapes similar?



Using a proportion grid to find the multiplicative relationship between the corresponding sides.

$\times \frac{3}{2}$

Small	Large
8	12
6	9

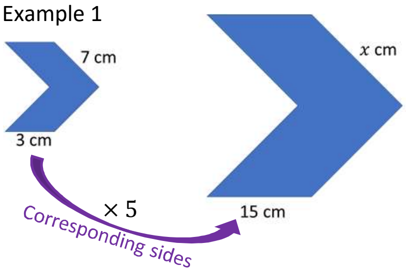
$\times \frac{3}{2}$

Because the multipliers are equal it is therefore similar

Information in similar shapes

If shapes are similar we can find missing sides using a proportion grid.

Example 1



× 5

Small	Large
3	15
7	x

× 5

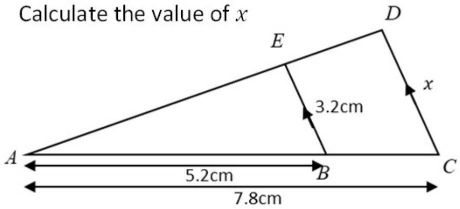
$$x = 7 \times 5$$
$$x = 35cm$$

Scale Factor is the multiplicative relationship between the two corresponding sides

Scale factor for this question is 5

Example 2

Calculate the value of x



$\times \frac{3}{2}$

Small	Large
5.2	7.8
3.2	x

$\times \frac{3}{2}$

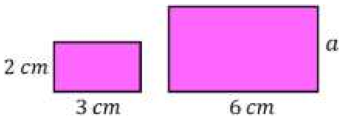
$$x = 3.2 \times \frac{3}{2}$$
$$x = 4.8cm$$

Scale factor for this question is $\frac{3}{2}$

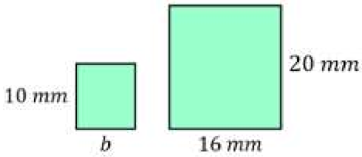
Your turn to practice

Find the missing sides of the following shapes:

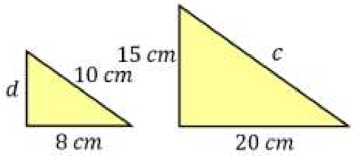
(a)



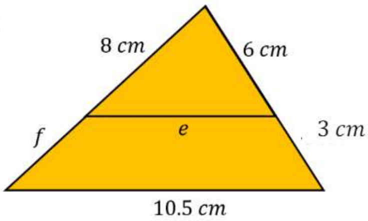
(b)



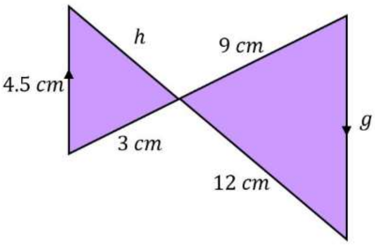
(c)



(d)



(e)



Answers

a) $a = 4cm$

b) $b = 8mm$

c) $c = 25cm, d = 6cm$

d) $e = 7, f = 4$

e) $g = 13.5cm, h = 4cm$

Best Buys

Keywords and Phrases:

Ratio and Proportion - describe the multiplicative relationship between two numbers.

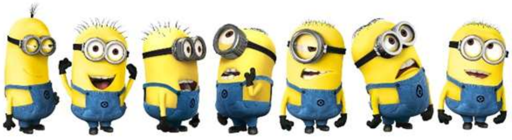
Unitary – Calculating how much one unit costs, to compare

Lowest common multiple – Lowest common timetable

Highest common factor – Listing all of the factors (numbers that multiply to make) which one is the highest.

Best buys

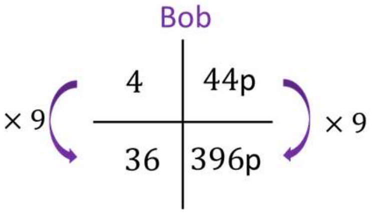
This concept is the most one of the most used every day piece of maths.
Comparing prices to ensure that you have the cheapest option available.
To do this, proportional reasoning is used (proportion grids)



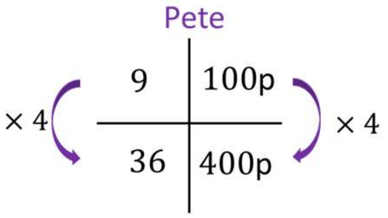
Example :
The minions go banana-shopping.
Bob finds 4 bananas cost 44p but Pete finds 9 bananas cost £1.
Which is the cheaper deal?

Option 1 – Finding the Lowest common multiple:

LCM of 4 and 9 = 36



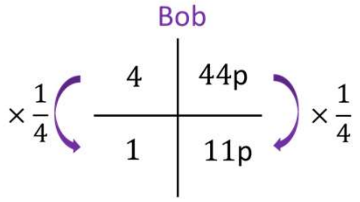
These a comparable because they both show what 36 bananas cost.



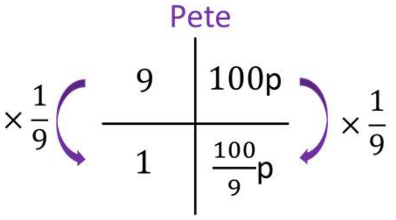
This is also called the unitary method, because you are finding what 1 banana will cost.

Option 2 – Finding the Highest common factor

HCF of 4 and 9 = 1



These a comparable because they both show what 1 banana cost.



Method selection – If this was non calculator, finding the LCM would be easier.
If calculator, you are most likely to use the HCF.

Your turn to practice

1)

Here are the costs of the same type of batteries in two shops.

Shop **A**	Shop **B**
Pack of 4 batteries £1.60	Pack of 6 batteries £2.70

Harry needs to buy at least 30 batteries.
He assumes that he has to buy batteries in whole packs.
Harry wants to buy the batteries as cheaply as possible from the same shop.

(a) Which shop should he buy the batteries from, shop **A** or shop **B**?
You must show all your working.

2)

A shop sells compost in 20 litre bags and in 40 litre bags.
One day the shop had two special offers for the compost.

2 bags for £3.50

3 bags for £9

Which offer is the better value for money?
You must show how you get your answer.

Answers

1) **Shop A** – 32 Batteries = £12.80, shop B 30 batteries for £13.50

2) **40 litre bag** – with comparative numbers, as per table:

Amount	20 litre bag	40 litre bag
120	£10.50	£9.00
80	£7.00	£6.00
40	£3.50	£3.00
20	£1.75	£1.50

Ratio - Unitary form

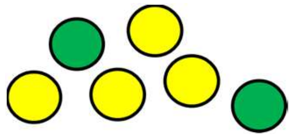
Keywords and Phrases:

Exchange rates – The ratio at which a unit of the currency of one country can be exchanged for that of another country

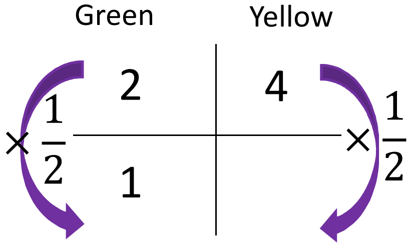
Unit ratio – A two part comparison where one part is a unit

Ratio and Proportion describe the multiplicative relationship between two numbers.

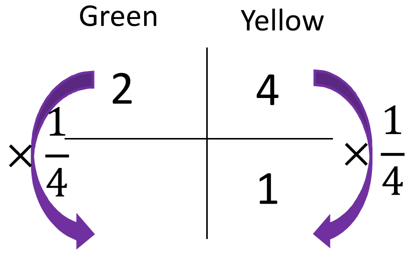
Example



Green to Yellow = 2:4
Yellow to Green = 4:2



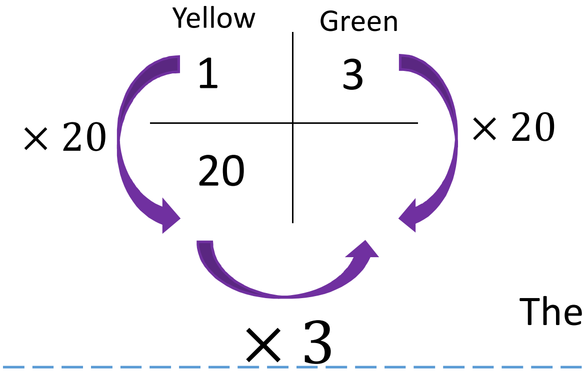
2:4 = 1:2



$2:4 = \frac{1}{2}:1$

When a ratio is written in unit form it gives the multiplier across

Yellow counters to green counters are in the ratio 1:3
How many yellow counters would there be if there were 20 yellow counters?



$$20 \times 3 = 60$$

Or

$$3 \times 20 = 60$$

There are 60 green counters

Your turn to practice. Write the ratios in the form 1:n using a proportion grid

1) $2:6$
2) $3:9$
3) $30:90$
4) $\frac{3}{5}:\frac{9}{5}$
5) $\frac{2}{17}:\frac{6}{17}$
6) $3x:9\mathrm{x}$
7) $3x:9$
8) $6:2$
9) $6x^2y:2xy^2$
10) $\frac{2x}{17y}:\frac{6x}{17y}$

Can you also write them in the form n:1

Answers:

1) 1:3
2) 1:3
3) 1:3
4) 1:3
5) 1:3
6) 1:3
7) $1:\frac{1}{x}$
8) $1:\frac{1}{3}$
9) $1:\frac{y}{6x}$
10) 1:3

1) $\frac{1}{3}:1$
2) $\frac{1}{3}:1$
3) $\frac{1}{3}:1$
4) $\frac{1}{3}:1$
5) $\frac{1}{3}:1$
6) $\frac{1}{3}:1$
7) $\frac{X}{3}:1$
8) 3:1
9) $\frac{3X}{Y}:1$
10) $\frac{1}{3}:1$

Exchange rates

Keywords and Phrases:

Exchange rates – The ratio at which a unit of the currency of one country can be exchanged for that of another country

Unit ratio – A two part comparison where one part is a unit (1)

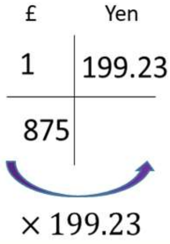
Ratio and Proportion describe the multiplicative relationship between two numbers.

Example 1:

Chloe and her family are travelling to Japan
The exchange rate is £1 : 199.23 yen

Chloe has £875 to convert into Japanese yen. How much yen will he get for her money?

Step 1 - Set up a proportion grid. Label the currencies and input the information from the question



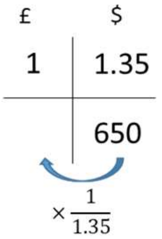
Step 2 – Clearly state your calculation

$$875 \times 199.23 = 174\,326.25\ Yen$$

Example 2:

When Bob is on holiday he sees an iPad that costs $650.
He knows that the iPad costs £500 at home.
Should he buy it? The exchange rate is still £1 : $1.35

Step 1 - Set up a proportion grid. Label the currencies and input the information from the question



Step 2 – Clearly state your calculation

$$650 \times \frac{1}{1.35} = 481.4814815 \ldots.$$

Step 3 - Round to actual money £481.48

Your turn to Practise

1 Euro = 109.74 Yen

a) A flight costs 325 euro. How much is this in Japanese Yen?
b) What is 275 Yen in Euros
c) The same TV costs 352 euros in France and 30 000 yen in Japan. Which is cheapest?

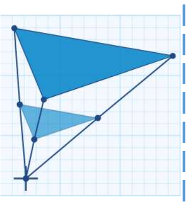
1 US Dollar = 73.2 Indian Rupees

a) A flight from the US to India costs $423. How much is this in Indian Rupees?
b) A mobile phone costs 20 000 Rupees How much is this in dollars?

Answers: a) 35665.50 yen b) 2.51 euros, c) 30 000 Yen, Qu2 a) 30963.60 Rupee, b) $273.22

Enlargement

Keywords and Phrases:



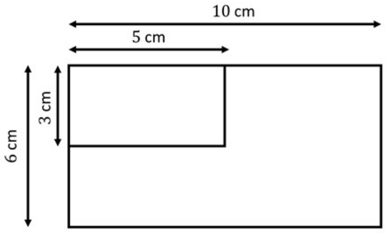
Enlargement – When you enlarge a shape it can get bigger or smaller… but it looks similar.

Length Scale factor – When you enlarge a shape, you will be multiplying each side a length scale factor.

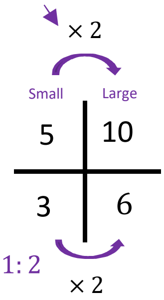
Similar shapes - Please refer to previous knowledge organiser on similar shapes.

Area scale factor – Is the length scale factor squared, and used to compare shapes similar areas.

Enlargement of shapes



The multiplier from small to large is 2



The ratio of the small to big rectangle is 1: 2

In this context we say that the **length scale factor of enlargement** is 2.

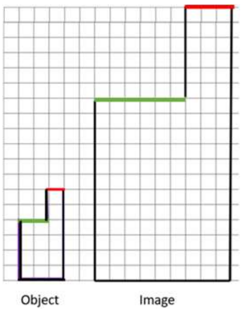
Each side length of the big rectangle is twice the length of the small rectangle.

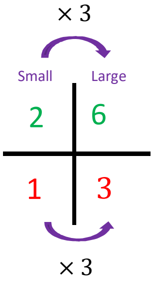
Enlarge this shape by a scale factor of 3:

Step 1 – Using the proportion grid, the multiplier is 3.

Step 2 – Consider every side length on the object you are enlarging, and multiply by 3.

Step 3 – Draw new image shape as below:





Enlargement and Area

When a shape is enlarged the area of the shape also changes, but not by the linear scale factor, but by the area scale factor.

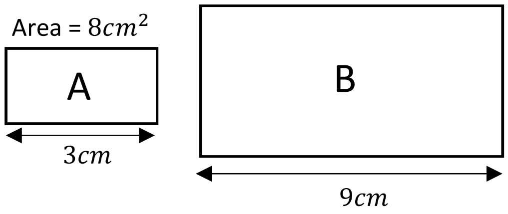
Length scale factor

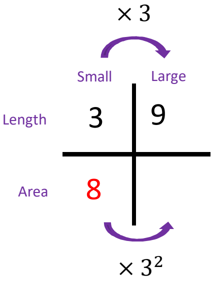
$$\text{Object} \xrightarrow{\times a} \text{Image}$$

Area scale factor

$$\text{Object} \xrightarrow{\times a^2} \text{Image}$$

Example: What is the area of rectangle B?



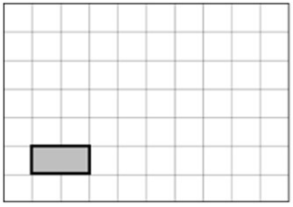


Area of B $= 8 \times 3^2$

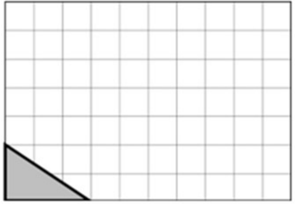
$= 72cm^2$

Your turn to practice:

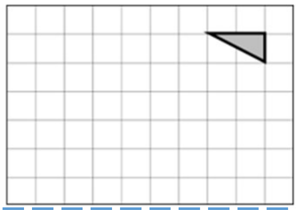
1) Enlarge by a SF of 2



2) Enlarge by a SF of 3

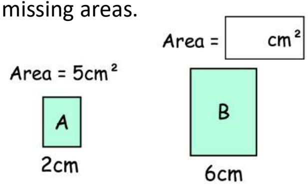


3) Enlarge by a SF of 4

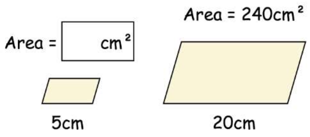


4) Each pair of shapes below are similar.
Find the missing areas.

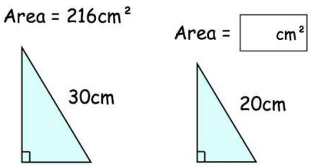
a)



b)

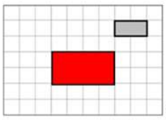


c)

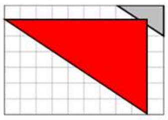


Answers

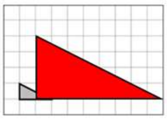
1)



2)



3)



4) a) $45cm^2$

b) $15cm^2$

c) $96cm^2$

Rational numbers

Keywords and Phrases:

Rational Numbers: Numbers that can be written as fractions, these include:

Integers (counting numbers)
e.g:

$$3 = \frac{3}{1}$$

Terminating Decimals
e.g:

$$0.32 = \frac{32}{100}$$

Recurring Decimals
e.g:

$$0.\dot{3} = 0.3333\ldots = \frac{1}{3}$$

Writing terminating decimals as fractions:

You could use a proportion grid:

Example: Convert $\frac{3}{8}$ to a decimal

$$\begin{array}{c|c} 3 & 375 \\ \hline 8 & 1000 \end{array} = \frac{375}{1000} = 0.375$$

Or division:

Example: Convert $\frac{5}{8}$ to a decimal

$$\begin{array}{r} 0.\,6\,2\,5 \\ 8\,\big)\overline{5.{}^{5}0{}^{2}0{}^{4}0} \end{array}$$

Recurring decimals to fractions:

$$\frac{1}{3} = 0.333333\ldots = 0.\dot{3}$$

This dot tells us that the 3 is recurring (repeats.)
Sometimes you might see $0.\overline{3}$ instead

$$\frac{4}{33} = 0.12121212\ldots = 0.\dot{1}\dot{2}$$

Both the 1 and 2 repeat.

$$\frac{1}{7} = 0.142857142857\ldots = 0.\underbrace{\dot{1}4285\dot{7}}$$

The whole group of numbers between the dots repeat.

When writing a recurring decimal as a fraction we need to find two multiples of the decimal that have the recurring part starting from the decimal point so that the recurring parts "line up."
One of these multiples could be the original recurring decimal.

Example 1:

Prove that $0.\dot{1}$ can be written as $\frac{1}{9}$

$$\text{Let } x = 0.\dot{1}$$

$$\begin{aligned} x &= 0.11111\ldots \\ &\quad \times 10 \mid \times 10 \\ 10x &= 1.11111\ldots \end{aligned}$$

$$\begin{aligned} 10x &= 1.11111\ldots \\ - \quad x &= 0.11111\ldots \\ \hline 9x &= 1 \\ &\quad \div 9 \mid \div 9 \\ x &= \frac{1}{9} \end{aligned}$$

Example 2:

Write $0.36\dot{7}$ as a fraction in it's simplest terms

$$\text{Let } x = 0.36\dot{7}$$

$$\begin{aligned} x &= 0.36777\ldots \\ &\quad \times 100 \mid \times 100 \\ 100x &= 36.77777\ldots \end{aligned}$$

$$\begin{aligned} x &= 0.36777\ldots \\ &\quad \times 1000 \mid \times 1000 \\ 1000x &= 367.77777\ldots \end{aligned}$$

$$\begin{aligned} 1000x &= 367.77777\ldots \\ - \quad 100x &= 36.77777\ldots \\ \hline 900x &= 331 \\ &\quad \div 900 \mid \div 900 \\ x &= \frac{331}{900} \end{aligned}$$

Example 3:

Write $0.\dot{1}\dot{7}$ as a fraction in it's simplest terms

$$\text{Let } x = 0.\dot{1}\dot{7}$$

$$\begin{aligned} x &= 0.171717\ldots \\ &\quad \times 100 \mid \times 100 \\ 100x &= 17.171717\ldots \end{aligned}$$

$$\begin{aligned} 100x &= 17.171717\ldots \\ - \quad x &= 0.171717\ldots \\ \hline 99x &= 17 \\ &\quad \div 99 \mid \div 99 \\ x &= \frac{17}{99} \end{aligned}$$

Your turn to practice:

Convert these fractions to decimals:

1) $\frac{1}{2}$
2) $\frac{1}{8}$
3) $\frac{3}{5}$
4) $\frac{3}{20}$
5) $\frac{1}{6}$
6) $\frac{1}{3}$
7) $\frac{4}{9}$
8) $\frac{7}{9}$
9) $\frac{1}{7}$
10) $\frac{5}{22}$

Convert these decimals to fractions

11) 0.3
12) 0.78
13) $0.\dot{4}$
14) $0.1\dot{4}$
15) $0.11\dot{4}$
16) $0.\dot{1}\dot{4}$
17) $0.1\dot{1}\dot{4}$

Answers

1) 0.5
2) 0.125
3) 0.6
4) 0.15
5) $0.1\dot{6}$
6) $0.\dot{3}$
7) $0.\dot{4}$
8) $0.\dot{7}$
9) $0.\dot{1}4285\dot{7}$
10) $0.2\dot{2}\dot{7}$
11) $\frac{3}{10}$
12) $\frac{39}{50}$
13) $\frac{4}{9}$
14) $\frac{13}{90}$
15) $\frac{103}{900}$
16) $\frac{14}{99}$
17) $\frac{113}{990}$

Surds

Keywords and Phrases:

Surd - When we can't simplify a number to remove a square root (or cube root etc) then it is a surd.

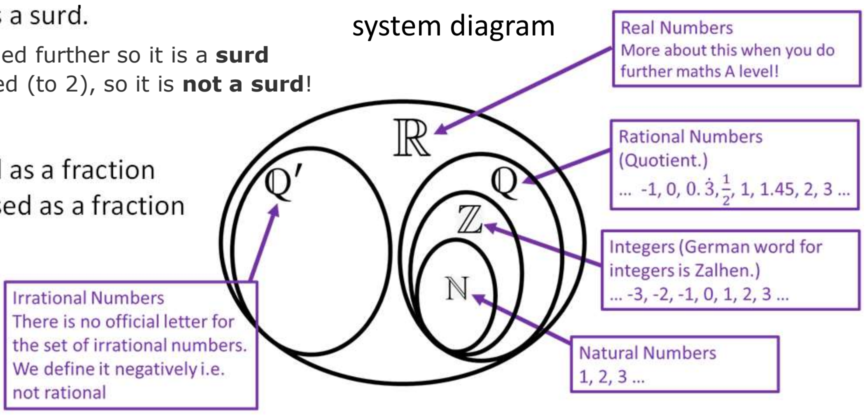
E.g: $\sqrt{2}$ (square root of 2) can't be simplified further so it is a **surd**
E.g: $\sqrt{4}$ (square root of 4) **can** be simplified (to 2), so it is **not a surd**!

Rational Numbers – **can** be expressed as a fraction
Irrational Numbers – **can't** be expressed as a fraction

$\pi = 3.14159 \ldots \ldots$
$\sqrt{2} = 1.4142 \ldots \ldots$
$\sqrt{3} = 1.73205 \ldots \ldots$

These are all irrational numbers, there are many more

The number system diagram



Real Numbers
More about this when you do further maths A level!

Rational Numbers (Quotient.)
... -1, 0, $0.\dot{3}$, $\frac{1}{2}$, 1, 1.45, 2, 3 ...

Integers (German word for integers is Zalhen.)
... -3, -2, -1, 0, 1, 2, 3 ...

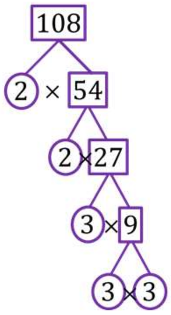
Natural Numbers
1, 2, 3 ...

Irrational Numbers
There is no official letter for the set of irrational numbers. We define it negatively i.e. not rational

Simplifying surds

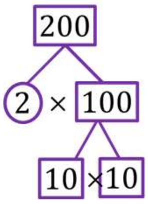
Using product of primes method to simplify a surd:

Example: Simplify $\sqrt{108}$



$\sqrt{108} = \sqrt{2 \times 2 \times 3 \times 3 \times 3}$
$= \sqrt{(2 \times 3) \times (2 \times 3) \times 3}$
$= \sqrt{6 \times 6} \times \sqrt{3}$
$= 6 \times \sqrt{3}$
$= 6\sqrt{3}$

Example: Simplify $\sqrt{200}$



$\sqrt{200} = \sqrt{10 \times 10 \times 2}$
$= \sqrt{10 \times 10} \times \sqrt{2}$
$= 10 \times \sqrt{2}$
$= 10\sqrt{2}$

The product of primes method will always work but if you can already identify a factor that is a square number you can use a more efficient method.

Adding and subtracting surds

When we add or subtract we need to have the same unit.
Simplify a surd as much as possible before you add or subtract

$2\sqrt{3} + 4\sqrt{5}$ We **can not** add these surds because they have different units

We **can** add these surds because they have different units $2\sqrt{5} + 4\sqrt{5} = 6\sqrt{5}$

Multiplying surds

Simplify each surd as much as possible before you multiply

In general:
$\sqrt{a} \times \sqrt{a} = a$

E.g:
$\sqrt{11} \times \sqrt{11} = 11$

$\sqrt{8} \times \sqrt{2} = 2\sqrt{2} \times \sqrt{2}$
$= 2 \times 2 \quad = 4$

$2\sqrt{75} \times 3\sqrt{27} = 2 \times 5\sqrt{3} \times 3 \times 3\sqrt{3}$
$= 2 \times 3 \times 5 \times 3 \times \sqrt{3} \times \sqrt{3}$
$= 270$

Your turn to practice:

1) Show that $\sqrt{900}$ is rational
2) Show that $\sqrt{600}$ is irrational

Simplify the following surds:

3) $\sqrt{12}$
4) $\sqrt{24}$
5) $\sqrt{48}$
6) $\sqrt{96}$
7) $\sqrt{75}$
8) $3\sqrt{7} + 2\sqrt{7} - 4\sqrt{7}$
9) $-8\sqrt{3} + 5\sqrt{5} + 9\sqrt{3} + 4\sqrt{7}$
10) $\sqrt{8} + \sqrt{18}$
11) $\sqrt{75} + \sqrt{300}$
12) $\sqrt{20} + \sqrt{500}$
13) $\sqrt{5} \times \sqrt{2}$
14) $\sqrt{7} \times \sqrt{11}$
15) $5\sqrt{2} \times 2\sqrt{2}$
16) $3\sqrt{7} \times 2\sqrt{7}$
17) $\sqrt{48} \times \sqrt{27}$

Answers
3) $2\sqrt{3}$
4) $2\sqrt{6}$
5) $4\sqrt{3}$
6) $4\sqrt{6}$
7) $5\sqrt{3}$
8) $\sqrt{7}$
9) $-5\sqrt{3} + 5\sqrt{5} + 4\sqrt{7}$
10) $5\sqrt{2}$
11) $15\sqrt{3}$
12) $12\sqrt{5}$
13) $\sqrt{10}$
14) $\sqrt{77}$
15) 20
16) 42
17) 36

Expanding Brackets with Surds

Keywords and Phrases:

Area Model for Expanding Brackets:

Using area model

	x	$+5$
x	x^2	$+5x$
$+4$	$+4x$	$+20$

Collect like terms $4x + 5x = 9x$

$$\equiv x^2 + 4x + 5x + 20$$

$$\equiv x^2 + 9x + 20$$

Simplest Form:

Remember some surds can be simplified

$$\sqrt{108} = \sqrt{2 \times 2 \times 3 \times 3 \times 3}$$

$$= \sqrt{(2 \times 3) \times (2 \times 3) \times 3}$$

$$= \sqrt{6 \times 6} \times \sqrt{3}$$

$$= 6 \times \sqrt{3}$$

$$= 6\sqrt{3}$$

Like Fractions you should always leave surds in their simplest form

Expanding a Single Bracket:

Use an area model to expand $\sqrt{3}(2 + \sqrt{3})$

	2	$+\sqrt{3}$
$\sqrt{3}$	$2\sqrt{3}$	$+3$

$\sqrt{3} \times \sqrt{3}$

$$\equiv 2\sqrt{3} + 3$$

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Expanding a Double Bracket:

Use an area model to expand $(5 + \sqrt{3})(2 + \sqrt{3})$

	2	$+\sqrt{3}$
5	10	$+5\sqrt{3}$
$+\sqrt{3}$	$+2\sqrt{3}$	$+3$

$$\equiv 10 + 5\sqrt{3} + 2\sqrt{3} + 3$$

$$\equiv 13 + 7\sqrt{3}$$

Use an area model to expand $(2 - 3\sqrt{5})(3 - 2\sqrt{5})$

	3	$-2\sqrt{5}$
2	6	$-4\sqrt{5}$
$-3\sqrt{5}$	$-9\sqrt{5}$	$+6 \times 5$

$-2 \times \sqrt{5} \times -3 \times \sqrt{5}$

$-2 \times -3 \times \sqrt{5} \times \sqrt{5}$

$$\equiv 6 - 9\sqrt{5} - 4\sqrt{5} + 30$$

$$\equiv 36 - 13\sqrt{5}$$

Your turn to practice:

Expand these Single Brackets:

(a) $\sqrt{2}(\sqrt{3} + 5)$ (b) $\sqrt{3}(\sqrt{5} + \sqrt{2})$ (c) $\sqrt{6}\,(2 - \sqrt{3})$ (d) $\sqrt{10}\,(5 + \sqrt{10})$

(e) $\sqrt{2}(\sqrt{18} - \sqrt{2})$ (f) $\sqrt{5}\,(3\sqrt{2} - \sqrt{5})$ (g) $2\sqrt{3}\,(3\sqrt{2} + \sqrt{3})$ (h) $4\sqrt{11}(5\sqrt{2} + 2\sqrt{11})$

Expand these **Double** Brackets

(a) $(2 + \sqrt{3})(1 + \sqrt{3})$ (b) $(\sqrt{2} + 5)(1 + \sqrt{2})$ (c) $(\sqrt{3} + 1)(\sqrt{3} + 4)$

(d) $(3 + \sqrt{5})(4 - \sqrt{5})$ (e) $(\sqrt{7} - 1)(\sqrt{7} - 1)$ (f) $(5 - \sqrt{3})(5 + \sqrt{3})$

(g) $(3 + \sqrt{2})(1 + \sqrt{3})$ (h) $(\sqrt{12} + \sqrt{3})(\sqrt{3} + 2)$ (i) $(4 - \sqrt{2})(3 + \sqrt{8})$

(j) $(\sqrt{7} + \sqrt{2})(\sqrt{8} + \sqrt{7})$ (k) $(1 + 2\sqrt{2})(2 - \sqrt{2})$ (l) $(3\sqrt{5} + 7)(2\sqrt{5} + 1)$

Corbett maths

SINGLE BRACKET ANSWERS

a)	$\sqrt{6} + 5\sqrt{2}$
b)	$\sqrt{15} + \sqrt{6}$
c)	$2\sqrt{6} - 3\sqrt{2}$
d)	$5\sqrt{10} + 10$
e)	4
f)	$3\sqrt{10} - 5$
g)	$6\sqrt{6} + 6$
h)	$20\sqrt{22} + 88$

DOUBLE BRACKET ANSWERS

a)	$5 + 3\sqrt{3}$
b)	$7 + 6\sqrt{2}$
c)	$7 + 5\sqrt{3}$
d)	$7 + \sqrt{5}$
e)	$8 - 2\sqrt{7}$
f)	22
g)	$3 + \sqrt{2} + 3\sqrt{3} + \sqrt{6}$
h)	$9 + 6\sqrt{3}$
i)	$8 + 5\sqrt{2}$
j)	$11 + 3\sqrt{14}$
k)	$-2 + 3\sqrt{2}$
l)	$37 + 17\sqrt{5}$

Functions

Keywords and Phrases:

Range – The values given from the output. Knows as dependent variables

Domain – The input values. Known as control variables.

Mapping diagrams – A representation to show the relationship between the input and output.

Function – How expressions relate to each other.

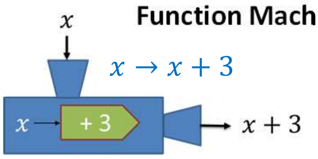
Function notation

$x \rightarrow 2x + 80$

This means that x maps to $2x + 80$
As an instruction we would say
multiply by 2 and then add 80

Function Machines

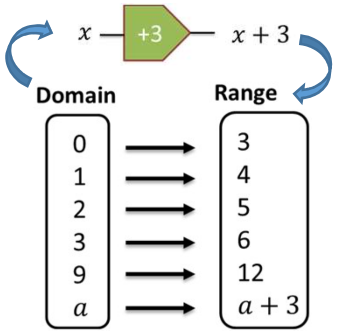
$x \rightarrow x + 3$



$x \rightarrow 2x + 80$

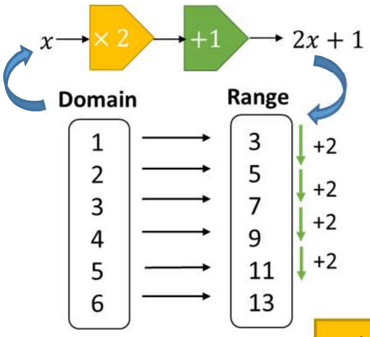
A function machine is how we create functions from our operations. Above are the two examples of functions machines you will see.

Mapping diagrams



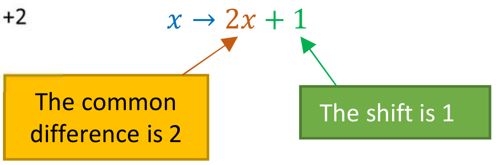
We use **mapping diagrams** to find different values from a given value.

$x \rightarrow 2x + 1$



If the domain is consecutive we can use our knowledge of sequences to understand the function.

The letters we use are different but the principle is the same.



Mapping diagrams

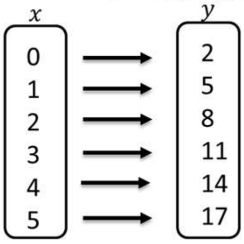
All functions can be plotted on a Cartesian grid. The mapping diagram helps to identify the coordinates.

On a graph the range is always y and the domain is always x.

Example: Draw the graph of $y = 3x + 2$ for values of x from 0 to 5

Step 1 – Draw a **function machine**

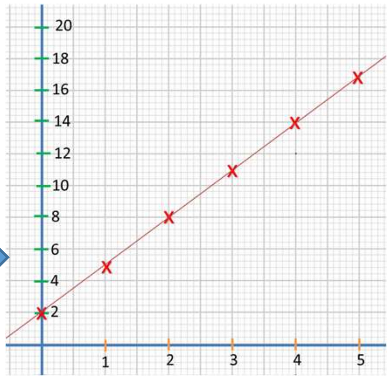
Step 2 – Given the domain values from 0 to 5 draw the **mapping diagram.**



Step 3 – We can write all of the pairs of numbers from our mapping diagram as **coordinates.**

(x, y)
(0,2)
(1,5)
(2,8)
(3,11)
(4,14)
(5,17)

Step 4 – Plot on a coordinate axis and join to create the graph

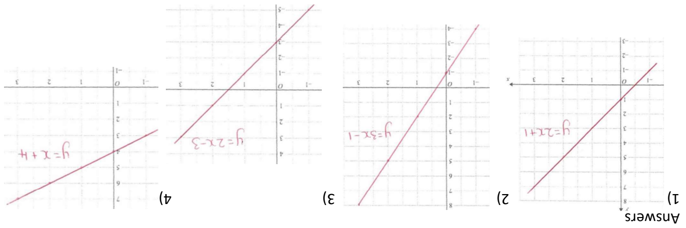


Your turn to practice:

Draw a mapping diagram for x values from -1 to 3, and plot the coordinates to create the graph:

1) $y = 2x + 1$
2) $y = 3x - 1$
3) $y = 2x - 3$
4) $y = x + 4$

Plot the on a graph,
y axis from 8 to -6,
x axis -1 to 3



Linear Graphs

Keywords and Phrases:

Linear graph – Straight line graph

Gradient – How steep the line is, the rate of change.

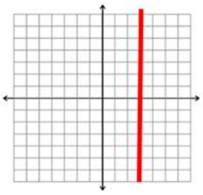
y-intercept – Where the lines crosses the y axis

$$y = mx + c$$

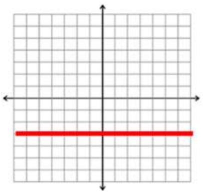
Gradient

y – intercept

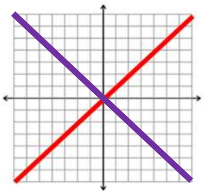
Equations of vertical, horizontal and diagonal lines



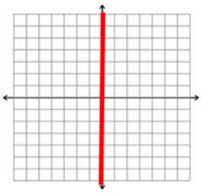
Vertical graph
$x = a$
a is the point where the line crosses the $x - ax$



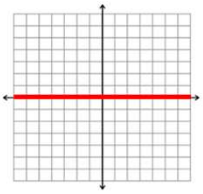
Horizontal graph
$y = a$
a is the point where the line crosses the y $- axis$



Diagonal graph
$y = x$
$y = -x$



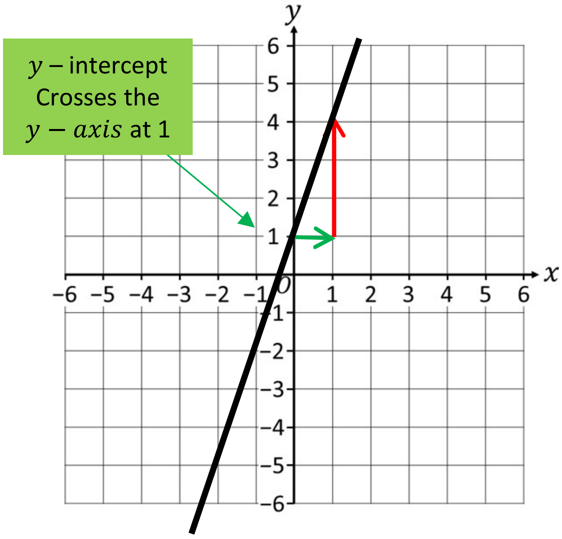
Vertical graph
$x = 0$
or
y – axis



Horizontal graph
$y = 0$
or
x – axis

Plotting straight line graphs from gradient and y-intercept

Example Plot $y = 3x + 1$



y – intercept Crosses the $y - axis$ at 1

$$y = 3x + 1$$

Gradient
The graph will increase by 3 for every 1 across

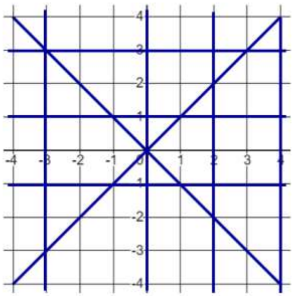
y – intercept
Crosses the $y - axis$ at 1

Difference in x	Difference in y
1	3

HINT
Always start the gradient at where the y-intercept.
As per the example.

Your turn to practice:

What are the equations of the following straight line graphs:

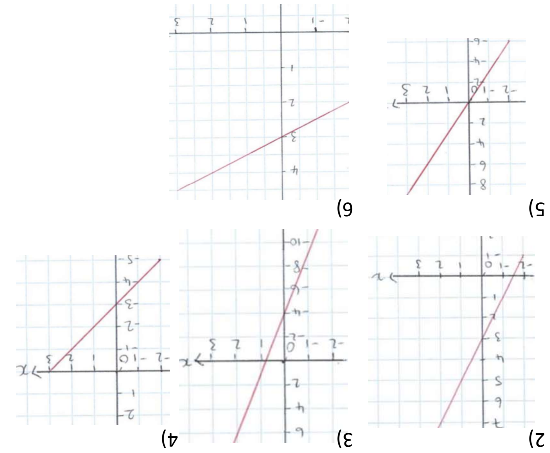


For each equation, draw its graph for values of x from −2 to 3.

1) $y = 2x + 3$
2) $y = 5x - 4$
3) $y = x - 3$
4) $y = 3x$
5) $y = \frac{1}{2}x + 3$

Answers

1) $x = -3, x = 2, y = 4, y = 3, y = 1, y = -1, y = x, y = -x$



Linear Graphs - Gradient

Keywords and Phrases:

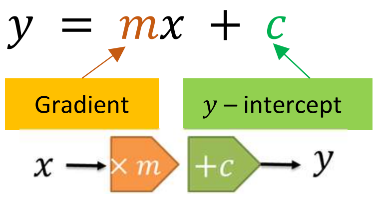
Linear graph – Straight line graph

Gradient – How steep the line is, the rate of change.

y-intercept – Where the lines crosses the y axis

Equations of Linear graphs

The equation $\boldsymbol{y = mx + c}$ is the general equation of any straight line. Where $\boldsymbol{m}$ is the **gradient** of the line (how steep the line is, the rate of change) and $\boldsymbol{c}$ is the $\boldsymbol{y}$**-intercept** (the point in which the line crosses the y-axis).



y and x are variables *m and C are constants*

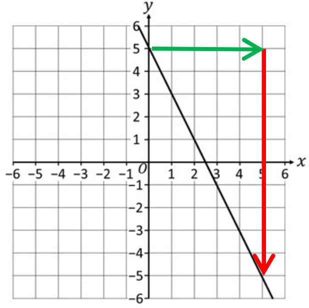
Gradient of a linear graph

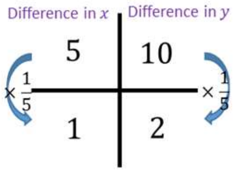
The gradient m is the rate at which graph increase or decrease for every **one unit** across.

You can use proportion grids to calculate the gradient, either from a graph or two coordinates.

Example:

Find the gradient of this line





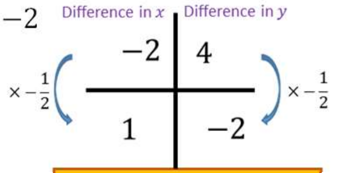
The gradient is 2

Example:

Find the gradient of a line passing through (0, 3) & (-2, 7)

Difference in $x = -2 - 0$
$= -2$

Difference in y $= 7 - 3$
$= 4$

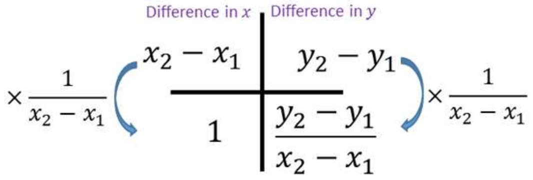


The gradient is -2

In general to calculate the gradient of any linear graph which passes through coordinates (x_1, y_1) & (x_2, y_2).

Difference in $x = x_2 - x_1$

Difference in y $= y_2 - y_1$



The gradient is $\frac{y_2 - y_1}{x_2 - x_1}$

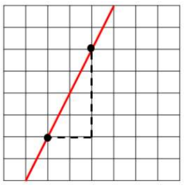
Your turn to practice:

What is the gradient (m) and y-intercept (c) of the following linear graphs:

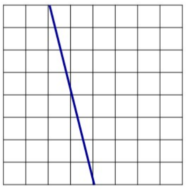
1) $y = 3x + 8$
2) $y = 2x - 7$
3) $y = 8 + 6x$
4) $2y = 6x - 8$
5) $3y = -x + 15$

Find the gradient of:

6)



7)



Find the gradient of each line passing through the coordinates:

8) (0, 0) & (2, 6)
9) (0, 3) & (2, 7)
10) (2, 2) & (4, 8)
11) (3, 0) & (5, –4)
12) (8, –5) & (6, –4)

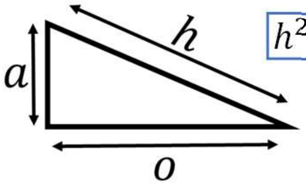
Answers

1) $m = 3, c = 8$
2) $m = 2, c = -7$
3) $m = 6, c = 8$
4) $m = 3, c = -4$
5) $m = -\frac{1}{3}, c = 5$
6) $m = 2$
7) $m = -4$
8) $m = 3$
9) $m = 2$
10) $m = 3$
11) $m = -2$
12) $m = -\frac{1}{2}$

Pythagoras

Keywords and Phrases:

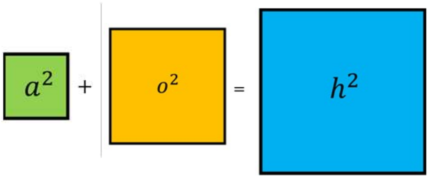
Pythagoras - Over 2000 years ago there was an amazing discovery about triangles:

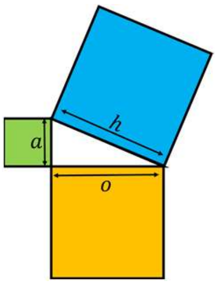


$$\boxed{h^2 = o^2 + a^2}$$

Pythagoras' theorem is only true for right angled triangles

When a triangle has a right angle (90°), and squares are made on each of the three sides. Then the biggest square has the ***exact same area*** *as the other two squares put together!*

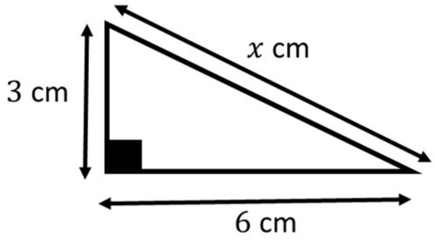


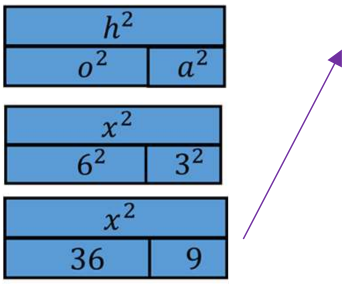


Using Pythagoras

Calculate the value of x

If there is no angle mentioned in the question it is probably a Pythagoras question





$x^2 = 36 + 9$

$x^2 = 45$

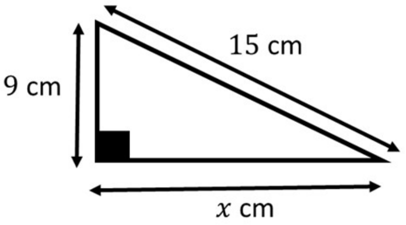
√ √

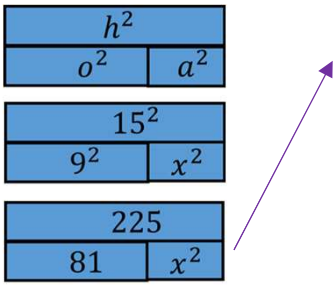
$x = \sqrt{45}$

$x = \sqrt{9 \times 5} \quad = 3\sqrt{5}cm$

Using Pythagoras

Calculate the value of x





$x^2 = 225 - 81$

$x^2 = 144$

√ √

$x = \sqrt{144}$

$x = 12cm$

Scan me

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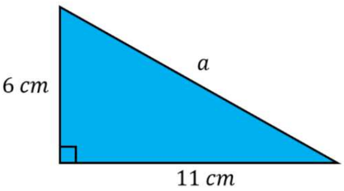
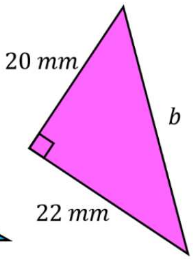
Your turn to practice:

1) Find the missing lengths.

(a)

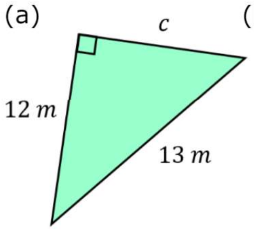


(b)

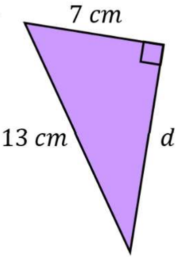


2) Find the missing lengths.

(a)

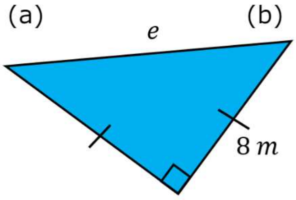


(b)

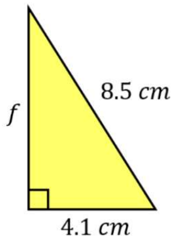


3) Find the missing lengths.

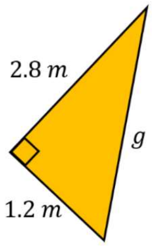
(a)



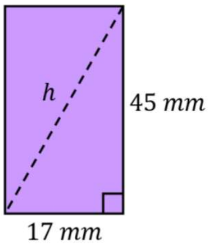
(b)



(c)



(d)



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Answers

1) a) 12.5cm b) 29.7mm 2) a) 5m b) 11.0cm 3) a) 11.3m b) 7.4cm c) 3.0m d) 48.1mm

Trigonometry

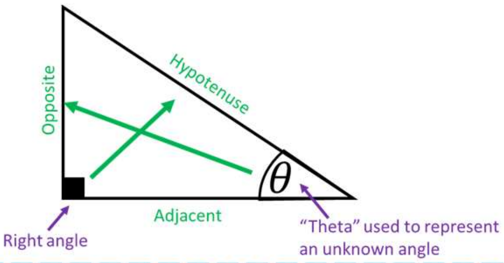
Keywords and Phrases:

Trigonometry - *(from Greek trigonon "triangle" + metron "measure")*
Trigonometry helps us find angles and distances, and is used a lot in science, engineering, video games, and more!

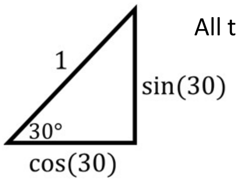
Adjacent - is always next to the angle that we are working with

Opposite - is always opposite the angle that we are working with

Hypotenuse – Is the longest side and is always opposite the right angle



The unit triangle



All trigonometry is based on this triangle at GCSE.

$\text{Sin}(\theta)$ is always **opposite** the angle
$\text{Cos}(\theta)$ is always **adjacent** to the angle

Exact trigonometry ratios

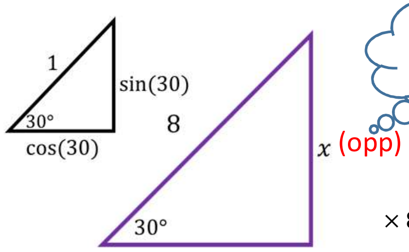
Trigonometry questions can appear on a non calculator paper.

You need to remember these values for your GCSE's.

	$\sin\theta$	$\cos\theta$
$\theta = 0°$	0	1
$\theta = 30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\theta = 45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\theta = 60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\theta = 90°$	1	0

Using the unit triangle to solve trigonometry questions:

Example 1: Using the unit triangle, create similar triangles



Our unknown is opposite so we use sin(30)

Unit triangle

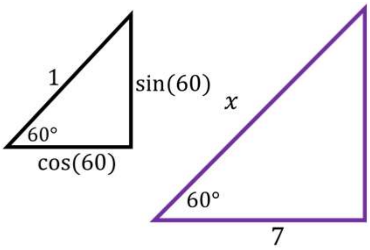
$$\times 8 \quad \begin{array}{c|c} 1 & \sin(30) \\ \hline 8 & x \end{array} \quad \times 8$$

Question

$$x = \sin(30) \times 8$$
$$x = \frac{1}{2} \times 8$$
$$x = 4cm$$

$$\sin(30) = \frac{1}{2}$$

Example 2: Using the unit triangle, create similar triangles



Unit triangle

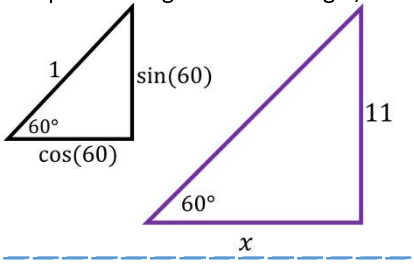
$$\times \frac{7}{\cos(60)} \quad \begin{array}{c|c} 1 & \cos(60) \\ \hline x & 7 \end{array} \quad \times \frac{7}{\cos(60)}$$

Question

$$x = 1 \times \frac{7}{\cos(60)}$$
$$x = 7 \div \frac{1}{2}$$
$$x = 14cm$$

$$\cos(60) = \frac{1}{2}$$

Example 3: Using the unit triangle, create similar triangles



Unit triangle

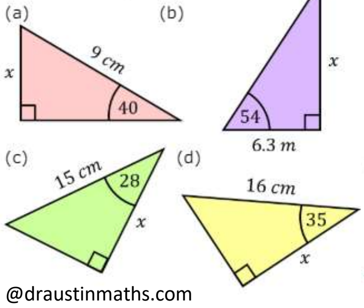
$$\times \frac{11}{\sin(60)} \quad \begin{array}{c|c} \sin(60) & \cos(60) \\ \hline 11 & x \end{array} \quad \times \frac{11}{\sin(60)}$$

Question

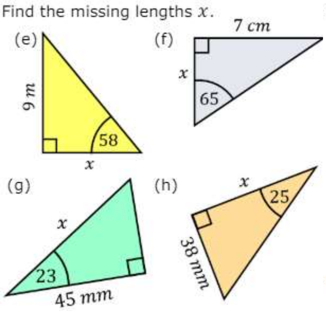
$$x = \cos(60) \times \frac{11}{\sin(60)}$$
$$x = \frac{1}{2} \times \frac{11}{\frac{\sqrt{3}}{2}}$$
$$x = \frac{1}{2} \times 11 \times \frac{2}{\sqrt{3}} \qquad x = \frac{11}{\sqrt{3}}$$

Your turn to practice: You will need a calculator for these questions:

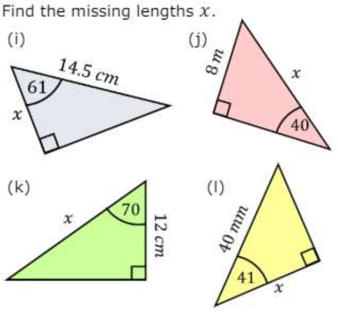
Find the missing lengths x.



Find the missing lengths x.



Find the missing lengths x.



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Answers

a) $x = 5.8cm$
b) $x = 58.7m$
c) $x = 13.2cm$
d) $x = 13.1cm$
e) $x = 5.6m$
f) $x = 3.3cm$
g) $x = 48.9mm$
h) $x = 81.5mm$
i) $x = 7.0cm$
j) $x = 12.4m$
k) $x = 35.1cm$
l) $x = 30.2mm$

Angles in Triangles

Keywords and Phrases:

$A\hat{B}C$ is the angle at B created by line segments AB and BC

Justify your reasoning – use your angle statements to explain your statements/equations

Diagonal – line joining 2 non-adjacent vertices

Bisect – cut exactly in half

Parallel lines – having the same direction

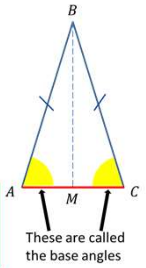
Opposite – across from, facing

Properties of Triangles

Angles in a triangle sum to 180°

Angles in an equilateral triangle are all 60°

Base angles in an isosceles triangle are equal

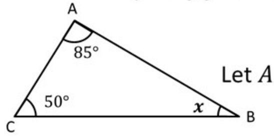


Triangle	Number of Equal Sides	Number of Equal Angles	Line of Symmetry (Y/N) How Many?	Can be Right-Angled
Equilateral	All 3 sides are equal	All 3 angles are equal (60°)	Yes 3 Lines of Symmetry	NO
Isosceles	2 sides are equal	2 angles are equal	Yes 1 Line of Symmetry	YES acute angles are 45°
Scalene	All sides are different	All angles are different	No	YES

Finding Angles in Triangles

$B\hat{A}C = 85°,\ B\hat{C}A = 50°$

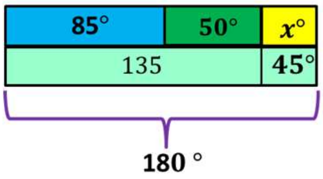
Find $A\hat{B}C$ and justify your reasoning



Let $A\hat{B}C = x$

Angles in a triangle sum to 180°

Using a bar model



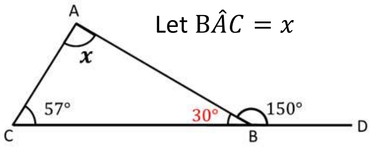
Using formal working

$$85 + 50 + x = 180$$
$$135 + x = 180$$
$$135 + x = 135 + 45$$
$$-135 \qquad -135$$
$$x = 45$$

Remember we were asked to find $A\hat{B}C$

$\underline{A\hat{B}C = 45°}$

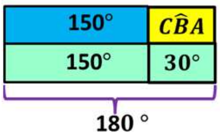
$D\hat{B}A = 150°,\ B\hat{C}A = 57°$ Find $B\hat{A}C$ and justify your reasoning



Let $B\hat{A}C = x$

Adjacent angles on a straight line sum to 180°

Using a bar model



OR

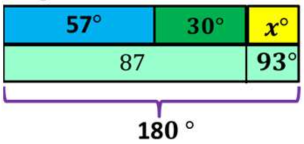
Using formal working

$$150 + C\hat{B}A = 180$$
$$-150 \qquad -150$$
$$C\hat{B}A = 30°$$

Add this to the diagram

Angles in a triangle sum to 180°

Using a bar model



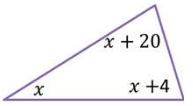
OR

Using formal working

$$57 + 30 + x = 180$$
$$87 + x = 180$$
$$87 + x = 87 + 93$$
$$-87 \qquad -87$$
$$x = 93°$$

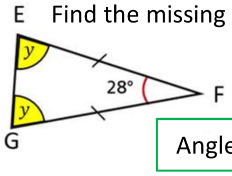
Remember we were asked to find $B\hat{A}C$

$B\hat{A}C = 93°$



Find the value of x

$$x + (x + 20) + (x + 4) = 180°$$
$$3x + 24 = 180°$$
$$3x + 24 = 156 + 24$$
$$-24 \qquad -24$$
$$3x = 156$$
$$\div 3 \qquad \div 3$$
$$x = 52°$$



Find the missing angles

$G\hat{E}F = E\hat{G}F$

Label these y

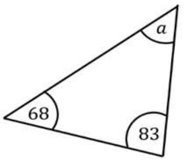
Base angles in an isosceles triangle are equal

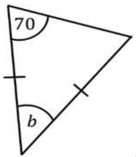
Angles in a triangle sum to 180°

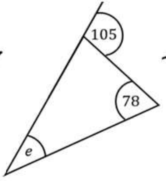
$$y + y + 28 = 180$$
$$2y + 28 = 180$$
$$2y + 28 = 152 + 28$$
$$-28 \quad | \quad -28$$
$$2y = 152$$
$$\div 2 \quad | \quad \div 2$$
$$y = 76°$$

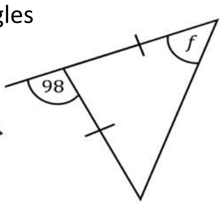
Practice questions

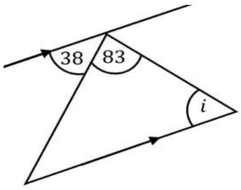
Find the missing angles

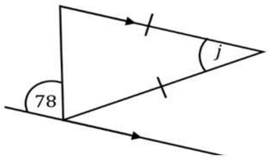












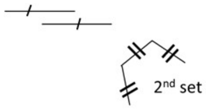
$a = 29°$
$b = 40°$
$e = 27°$
$f = 49°$
$i = 59°$
$j = 24°$

Angles in Quadrilaterals

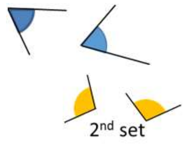
Properties of Quadrilaterals

Notation

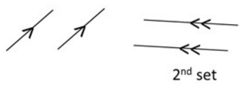
Equal length sides



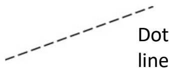
Equal angles



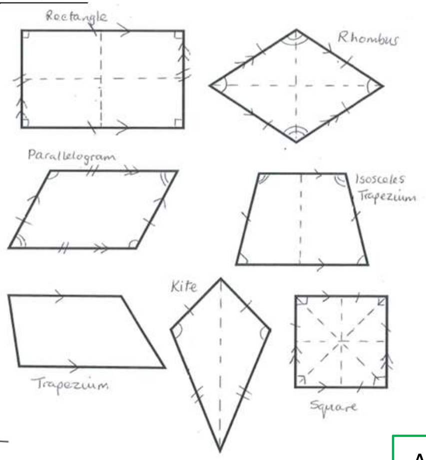
Parallel sides



Line of symmetry



Dotted line

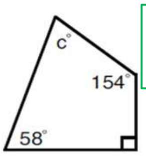


	Parallelogram	Rectangle	Square	Rhombus	Kite	Trapezium	Isosceles Trapezium
Are both pairs of opposite angles equal?	Yes	Yes	Yes	Yes	No Only one pair	No	No
Are *all* angles equal?	No	Yes	Yes	No	No	No	No
Are both pairs of opposite sides parallel?	Yes	Yes	Yes	Yes	No	No Only one pair	No Only one pair
Are both pairs of opposite sides equal?	Yes	Yes	Yes	Yes	No	No	No Only one pair
Are *all* sides equal?	No	No	Yes	Yes	No	No	No
Are the two diagonals equal?	Yes	Yes	Yes	No	No	No	Yes
Do the diagonals bisect each other?	Yes	Yes	Yes	Yes	No Shorter one is bisected	No	No
Do the diagonals intersect at right angles?	No	No	Yes	Yes	Yes	No	No
How many lines of symmetry are there?	0	2	4	2	1	0	1

Angle statements

- Angles in a quadrilateral sum to 360°
- One pair of opposite angles in a kite are equal
- Opposite angles in a parallelogram/rhombus are equal
- Co-interior angles in a trapezium sum to 180°
- Base angles in an isosceles trapezium are equal

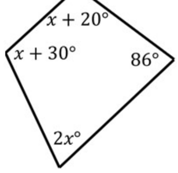
Finding Angles in Quadrilaterals



Angles in a quadrilateral sum to 360°

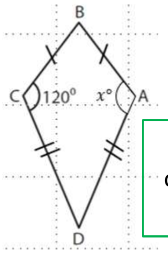
$$58 + 154 + 90 + c = 360°$$
$$302 + c = 360°$$
$$\overset{0}{\cancel{302}} + c = \overset{0}{\cancel{302}} + 58°$$
$$-302 \qquad -302$$
$$c = 58°$$

Find the value of x



angles in a quadrilateral sum to 360°

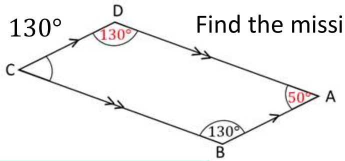
$$86 + (x + 20) + (x + 30) + 2x = 360$$
$$136 + 4x = 360$$
$$\overset{0}{\cancel{136}} + 4x = \overset{0}{\cancel{136}} + 224$$
$$-136 \qquad -136$$
$$4x = 224$$
$$\div 4 \qquad \div 4$$
$$x = 56°$$



$x = 120°$

One pair of opposite angles in a kite are equal

$A\hat{B}C = 130°$



Opposite angles in a parallelogram are equal

$A\hat{D}C = 130°$

Find the missing angles in this quadrilateral

Co-interior Angles sum to 180°

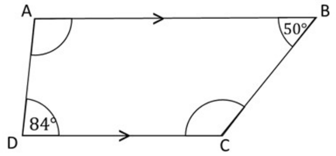
$$D\hat{A}B + A\hat{B}C = 180°$$
$$D\hat{A}B + 130 = 180$$
$$D\hat{A}B + \overset{0}{\cancel{130}} = 50 + \overset{0}{\cancel{130}}$$
$$-130 \qquad -130$$
$$D\hat{A}B = 50°$$

Add this to the diagram

Opposite angles in a parallelogram are equal

$D\hat{C}B = 50°$

Find the missing angles in this quadrilateral



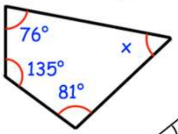
Co-interior Angles sum to 180°

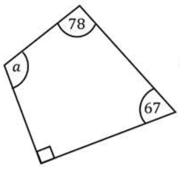
$$D\hat{A}B + A\hat{D}C = 180$$
$$D\hat{A}B + 84 = 180$$
$$D\hat{A}B + \overset{0}{\cancel{84}} = 96 + \overset{0}{\cancel{84}}$$
$$-84 \qquad -84$$
$$D\hat{A}B = 96°$$

$$B\hat{C}D + C\hat{B}A = 180$$
$$B\hat{C}D + 50 = 180$$
$$B\hat{C}D + \overset{0}{\cancel{50}} = 130 + \overset{0}{\cancel{50}}$$
$$-50 \qquad -50$$
$$B\hat{C}D = 130°$$

Practice questions

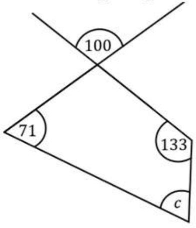
Find the missing angles

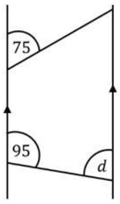




$a = 125°$

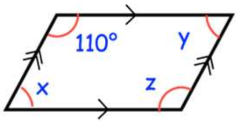
$x = 68°$





$d = 85°$

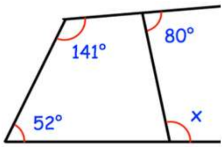
$c = 56°$

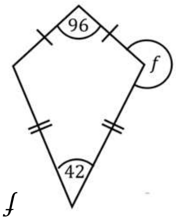


$z = 110°$

$y = 70°$

$x = 70°$





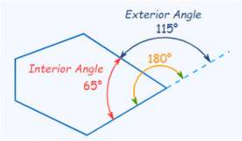
$f = 249°$

$x = 113°$

Angles in Polygons

Keywords and Phrases:

Polygon – a 2-D shape created when adjacent vertices are connected by line segments (straight lines)
Irregular polygon – a polygon that does not have all sides equal and all angle equal
Regular polygon – a polygon where all sides are equal and all angles are equal
Interior angle – the angle created between 2 adjacent sides, inside the shape
Exterior angle – the angle created between a side and the line extended from the next side



The interior angle and the exterior angle sum to 180°

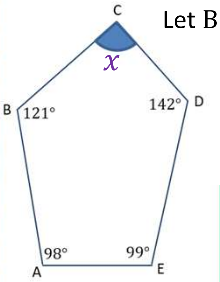
number of sides → - 2 → X 180 → sum of the interior angles

The interior angles of an n −sided polygon sum to $180(n-2)°$

The sum of the exterior angles for **<u>any polygon</u>** is 360°

Finding Angles in Polygons

Calculate the size of angle $B\hat{C}D$



Let $B\hat{C}D = x$

5 sides (3 triangles)

Sum of interior angles

$3 \times 180 = 540°$

$142 + 99 + 98 + 121 + x = 540$

$\cancelto{0}{460} + x = \cancelto{0}{460} + 80$

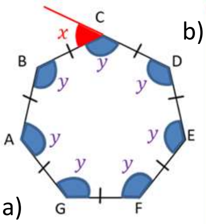
$-460 \qquad -460$

$x = 80°$

$B\hat{C}D = 80°$

$ABCDEFG$ is a regular heptagon

Calculate the size of each a) exterior angle b) interior angle



a)

7 equal exterior angles

Exterior angles sum to 360°

$7x = 360°$

$\div 7 \qquad \div 7$

<u>$x = 51.4°$ (1dp)</u>

b)

Exterior angle + interior angle = 180°

$51.4 + y = 180$

$\cancelto{0}{51.4} + y = \cancelto{0}{51.4} + 128.6$

$-51.4 \qquad -51.4$

$y = 128.6°$

Or

7 sides (5 triangles)

Interior angles sum to

$5 \times 180 = 900°$

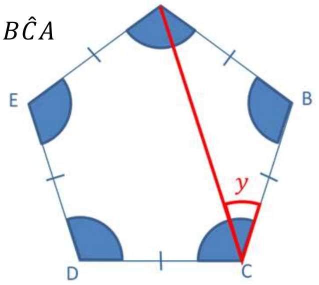
If each angle is y

$7y = 900$

$\div 7 \qquad \div 7$

<u>$y = 128.6°$(1dp)</u>

Find $B\hat{C}A$



How to plan your solution

This is a regular pentagon – all angles are equal
ABC is an isosceles triangle so base angles are equal
Find $A\hat{B}C$, then find angle $B\hat{C}A$

5 sides (3 triangles)

Interior angles sum to

$3 \times 180 = 540°$

If each angle is x

$5x = 540$

$\div 5 \qquad \div 5$

$x = 108°$

ABC is an isosceles triangle

$B\hat{A}C = B\hat{C}A = y$

$108 + y + y = 180$

$\cancelto{0}{108} + 2y = \cancelto{0}{108} + 72$

$-108 \qquad -108$

$2y = 72$

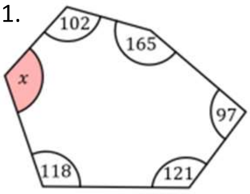
$\div 2 \qquad \div 2$

$y = 36°$

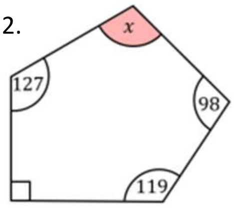
<u>$B\hat{C}A = 36°$</u>

Practice questions

1.



2.

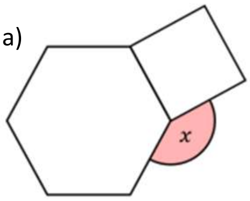


3. Work out the sum of the interior angles of a polygon with
   a) 14 sides b) 45 sides

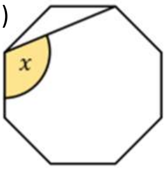
4. Calculate the i) exterior angle and ii) interior angle of a regular hexagon

5. All the polygons are regular. Find the value for x in each diagram

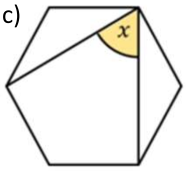
a)



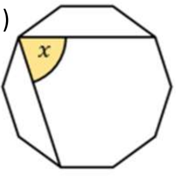
b)



c)



d)



Answers 1. 117° 2. 106° 3. a) 2160° b) 7740° 4. i) 60° ii) 120° 5. a) 150° b) 112.5° c) 60° d) 72°

Graphing Simultaneous Equations

Keywords and Phrases:

Simultaneous means “at the same time”.
Linear graph – Straight line graph
Gradient – How steep the line is, the rate of change.
y-intercept – Where the lines crosses the y axis

$$y = mx + c$$

Gradient

y – intercept

Solving simultaneous equations graphically:

When straight lines cross graphically the coordinate where the lines intersect is the **simultaneous solution**

Example: Solve the simultaneous equation

$y = 3x - 4$
$y = 4 - 2x$

Difference in x	Difference in y
1	$\frac{1}{2}$
2	1

The gradient is $\frac{1}{2}$ make this easier to plot by making the ratio whole numbers

Step 1 – Plot both lines using the gradient and y-intercept method.

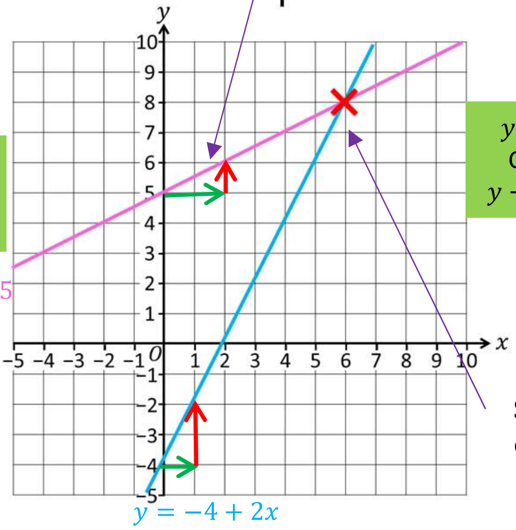
$$y = \frac{1}{2}x + 5$$

Gradient
The graph will increase by $\frac{1}{2}$ for every 1 across

Gradient = $\frac{1}{2}$

y – intercept
Crosses the $y - axis$ at $+5$

$y = \frac{1}{2}x + 5$



$$y = -4 + 2x$$

y – intercept
Crosses the $y - axis$ at -4

Gradient
The graph will decrease by 2 for every 1 across

Gradient = $+2$

Step 2 – Find the coordinates of where the two lines cross

(6, 8)

Your turn to practice:

Solve the following equations graphically:

1) $y = x + 5$
$y = 2x + 1$

2) $y = x + 4$
$y = 3x - 2$

3) $y = -x + 6$
$y = x - 4$

4) $y = 10 - x$
$y = x - 1$

5) $2y = 6 - x$
$y = 2x - 2$

6) $y = -2x + 8$
$y = \frac{1}{2}x - 2$

Answers
1) (4,9)
2) (3,7)
3) (5,1)
4) (5.5,4.5)
5) (2, 2)
6) (4,0)

Solving Simultaneous Equations

Keywords and Phrases:

Simultaneous means "at the same time".

In this topic it means that the solutions are true for both equations "at the same time".

Difference – means "subtraction".

To find the difference between the equations we subtract one from the other.

Sum – means "addition"

To solve these we will create zero pairs, then add the equations together to eliminate them

Direction – whether a number is positive or negative

Magnitude – the size of the number (regardless of direction)

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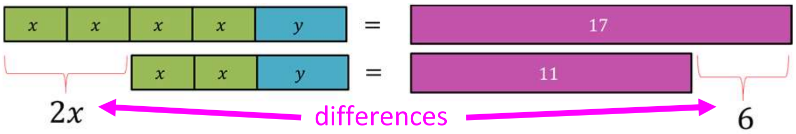
Solving using the "Difference" approach (when there are equal variables):

Step 1 find the difference between the left hand side and the right hand side

$4x + y = 17$

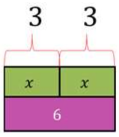
$2x + y = 11$



The "y" variables are equal in direction and magnitude

Step 2: equate the differences

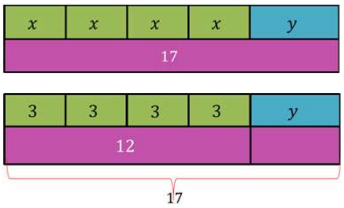
$$2x = 6$$



Step 3: solve to find the first variable

$x = 3$

Step 4: substitute into one of the original equations

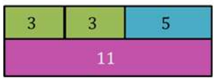


Step 5: solve for the other variable

$y = 5$

Step 6: CHECK! Using the other original equation

$2x + y = 11$



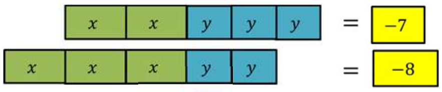
We know we are right!

$x = 3$ and $y = 5$

Solving using the "Difference" approach (when the variables are not equal):

$2x + 3y = -7$

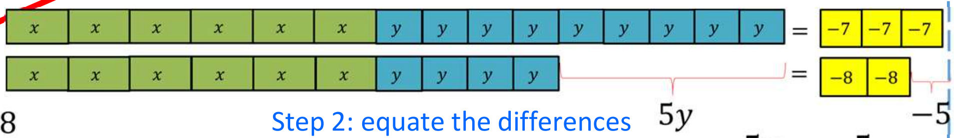
$3x + 2y = -8$



Step 1: Use the LCM of 2 and 3 to equalise the "x" variables

$$2x + 3y = -7 \xrightarrow{x3} 6x + 9y = -21$$
$$3x + 2y = -8 \xrightarrow[x2]{} 6x + 4y = -16$$

Equal "x" variables



Step 2: equate the differences

$5y = -5$

$y = -1$

Step 3: substitute and solve for x (as above)

$$3x + 2y = -8$$
$$3x - 2 = -8$$
$$+2 \quad +2$$
$$3x = -6$$
$$\div 3 \quad \div 3$$

$x = -2$

Step 4: CHECK! Using the other original equation

Your turn to practise:

Q1 Solve:

$x + 2y = 8$

$3x + 2y = 12$

Q2 Solve:

$x + y = 7$

$3x + y = 17$

Q3 Solve:

$x - y = 7$

$2x - y = 12$

Q4 Solve:

$4x - y = 10$

$3x - y = 8$

Q5 Solve:

$x + 2y = 13$

$2x + 3y = 22$

Q6 Solve:

$3x + 2y = 20$

$2x + y = 11$

Q7 Solve:

$3x + 4y = 1$

$2x + 5y = 3$

Q8 Solve:

$4x - 5y = 17$

$3x - 2y = 11$

Answers:

1. $x = 4, y = 2$
2. $x = 5, y = 2$
3. $x = 5, y = -2$
4. $x = 2, y = -2$
5. $x = 5, y = 4$
6. $x = 2, y = 7$
7. $x = -1, y = 1$
8. $x = 3, y = -1$

Solving Simultaneous Equations

Keywords and Phrases:

Simultaneous means “at the same time”.
In this topic it means that the solutions are true for both equations “at the same time”.
Difference – means “subtraction”.
To find the difference between the equations we subtract one from the other.
Sum – means “addition”
To solve these we will create zero pairs, then add the equations together to eliminate them
Direction – whether a number is positive or negative
Magnitude – the size of the number (regardless of direction)

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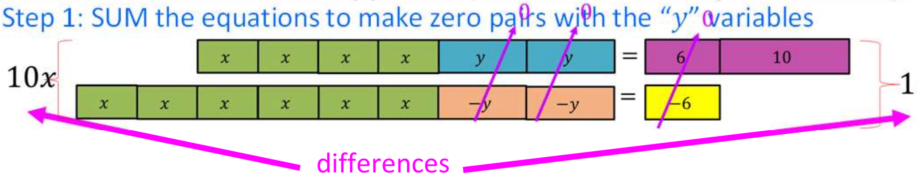
Solving using the “ZERO PAIRS and Sum” approach (when there are equal variables):

$4x + 2y = 16$
$6x - 2y = -6$

The “y” variables are EQUAL in magnitude but OPPOSITE in direction

Step 1: SUM the equations to make zero pairs with the “y” variables



Step 2: equate the sums

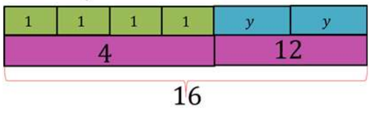
$10x = 10$

Step 3: solve to find the first variable

$x = 1$

Step 4: substitute into one of the original equations

$4x + 2y = 16$



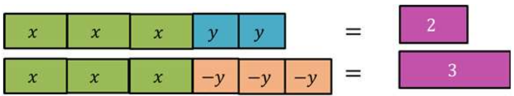
Step 5: solve for the other variable

$2y = 12$
$y = 6$

Step 6: CHECK! Using the other original equation

Solving using the “ZERO PAIRS and Sum” approach (when the variables are not equal):

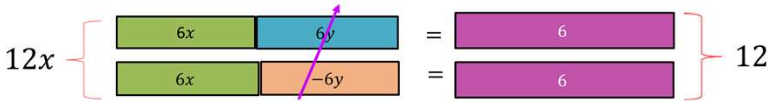
$2x + 2y = 2$
$3x - 3y = 3$



Step 1: Use the LCM of 2 and 3 to equalise the “y” variables as they are opposite in direction

$2x + 2y = 2 \xrightarrow{\times 3} 6x + 6y = 6$
$3x - 3y = 3 \longrightarrow 6x - 6y = 6$

Equal magnitude “y” variables



Step 2: equate the sums

$12x = 12$
$x = 1$

Step 3: substitute and solve for x (as above)

$2x + 2y = 2$
$\overset{0}{\cancel{2}} + 2y = \overset{0}{\cancel{2}}$
$-2 \qquad -2$
$2y = 0$
$y = 0$

Step 4: CHECK! Using the other original equation

Your turn to practise:

Q1 Solve:
$2x + y = 36$
$x - y = 9$

Q2 Solve:
$9x - 4y = 19$
$4x + 4y = 20$

Q3 Solve:
$3x + 3y = 54$
$2x - 2y = 16$

Q4 Solve:
$3x + 2y = 23$
$2x - y = 6$

Q5 Solve:
$6x + 3y = 45$
$2x - 2y = 12$

Q6 Solve:
$2x + 2y = 14$
$5x - 3y = 19$

Q7 Solve:
$5x + 2y = 38$
$2x - 3y = 19$

Q8 Solve:
$3x + 3y = -6$
$4x - 4y = -24$

Answers:
1. $x = 15, y = 6$ 2. $x = 3, y = 2$ 3. $x = 14, y = 6$ 4. $x = 5, y = 4$ 5. $x = 7, y = 1$ 6. $x = 5, y = 2$ 7. $x = 8, y = -1$ 8. $x = -4, y = 2$

Probability

Keywords and Phrases:

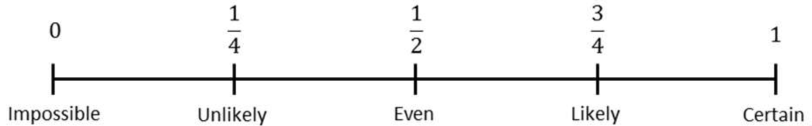
Probability scale - The **probability scale** is a number line from 0 to 1 where we can place the probability of events occurring. Events can range from impossible to certain.

Probability notation – You write the probability of red happening like this: $P(Red)$ ……

Probabilities are acceptable as fractions, decimals or percentages.

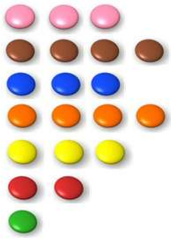
Probability scale:

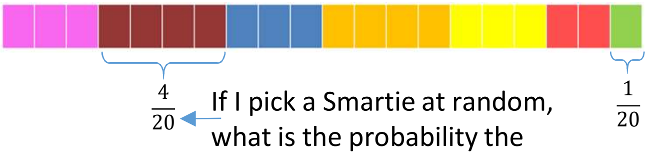
This is an example of the general probability scale, but there are far more possibilities that these:



Example:

There are 20 smarties, these can be represented as a bar model:



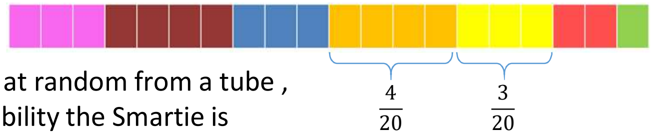


If I pick a Smartie at random, what is the probability the Smartie is brown?

P(Brown) = $\frac{4}{20}$

If I pick a Smartie at random from a tube, what is the probability the Smartie is green?

P(Green) = $\frac{1}{20}$



If I pick a Smartie at random from a tube , what is the probability the Smartie is orange or yellow?

P(Orange or Yellow) = $\frac{4}{20} + \frac{3}{20}$

$= \frac{7}{20}$

Probability Distribution

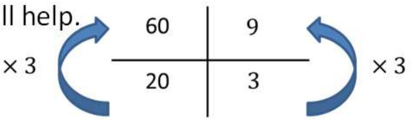
We can show all of the probability results from the smarties tube in a probability distribution table:

Smartie Colour x	Pink	Brown	Blue	Orange	Yellow	Red	Green
Probability $P(X = x)$	$\frac{4}{20}$	$\frac{4}{20}$	$\frac{3}{20}$	$\frac{4}{20}$	$\frac{3}{20}$	$\frac{2}{20}$	$\frac{1}{20}$

All probabilities add to 1, because it is certain that you will pick a smartie out of the tube.

If I pick out a Smartie and replace it 60 times estimate how many times I would pick a blue Smartie?

P(Blue) = $\frac{3}{20}$ So this is calculating $\frac{3}{20}$ of 60, therefore a proportion grid will help.

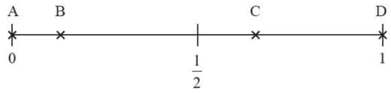


You could pick a blue out 9 times

Your turn to practice:

Here is a probability scale.

It shows events A, B, C and D.



1) Write down the letter of the event that is certain
2) Write down the letter of the event that unlikely.

3) There are 12 counters in a bag
3 red counters
1 blue counter
2 yellow counters
The rest are green

Sam takes at random a counter from the bag

What is the probability that the counter is yellow or green.

4) The table shows the probabilities a counter picked from a bag will be red, will be blue and will be yellow.

Colour	Red	Blue	Yellow	Green
Probability	0.2	0.4	0.3	

Complete the table.

5) 120 counters are picked randomly and replaced. Calculate an estimate for the amount of times the counter picked is blue.

Answers

1) D 2) B 3) $\frac{2}{3}$ 4) P(Green) = 0.1 5) 48 blue counters

Two way tables and frequency trees

Keywords and Phrases:

Two way tables – Two-way tables are a way of sorting data so that the frequency of each category can be seen quickly and easily.

Frequency trees - A frequency tree can be used to record and organise information given as frequencies. This can then be used to calculate probabilities.

Two way tables: Two way tables are a way of organising information from a long list.

Example: There are 170 students in year 7 at a school. Total of the two way table

All of these students either walk to school, get the bus to school or cycle to school. Helps set up table

82 of the students are boys.

33 of the students get the bus to school.

19 of the 41 students that walk to school are boys.

56 girls cycle to school.

Copy and complete the two way table.

Follow the rest of the instructions adding info to the table.

	Walk	Bus	Cycle	Total
Boys	19	7	56	82
Girls	22	26	40	88
Total	41	33	96	170

A student is chosen at random, calculate the probability that the student is a boy who cycled to school

$$P(boy\ who\ cycled) = \frac{56}{170}$$

56 boys cycled

Out of a total of 170 students

Frequency Trees: Frequency trees are a way of organising information

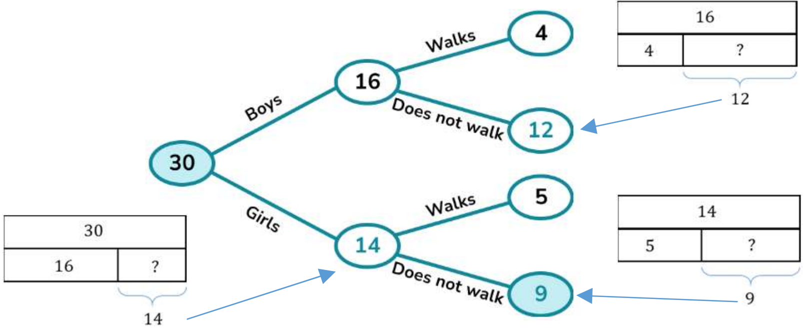
Example

A class has 30 children.

There are 16 boys.

4 boys and 5 girls walk to school.

Show this on a frequency tree



A child is chosen at random, calculate the probability that the child is a boy who walks to school

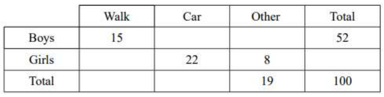
$$P(boy\ who\ walk\ \ )\quad \frac{4}{30}$$

4 boys walked

Out of a total of 30 children

Your turn to practice:

1) The two way table gives information about how 100 students travelled to school.

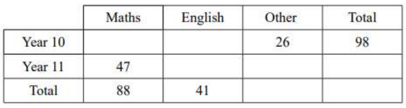


	Walk	Car	Other	Total
Boys	15			52
Girls		22	8	
Total			19	100

One of the students is picked at random.

Write down the probability they walk to school.

2) The two way table gives information about the favourite subject of 200 students.



	Maths	English	Other	Total
Year 10			26	98
Year 11	47			
Total	88	41		

One of the students is picked at random.

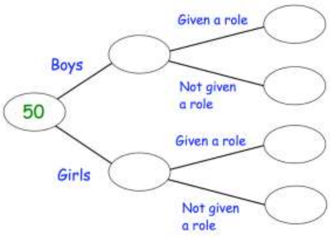
Write down the probability they are a year 10 student whose favourite subject is English.

3) 50 children audition for the school play.

18 of the children are boys.

15 children were given a role in the play.

8 girls were given a role in the play.



Complete the frequency tree.

What fraction of the boys were given a role in the play?

Answers

1) $\frac{33}{100}$

2) $\frac{31}{200}$

3) $\frac{7}{18}$

Probability – Venn Diagrams

Keywords and Phrases:

Venn diagram - A **Venn diagram** is a diagrammatic representation of **two or more sets**

Set notation -

ξ - The universal set, this set includes all the data.

$A \cap B$ - The intersection of set A and set B. The data that is common in both sets

$A \cup B$ - The union of set A and set B. All the data in both sets

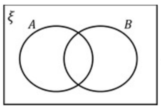
∈ - An element. One of the pieces of data in a set.

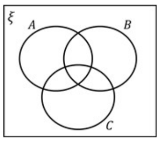
∉ - Not an element.

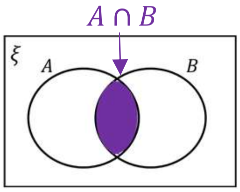
P(A) - Probability set A will occur.

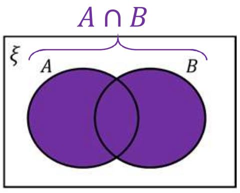
P(A') - Probability set A will **not** occur.

∅ - A null set. A set with no data in it.









Venn diagrams

Given that $\xi = \{$Natural numbers between 1 and 20$\}$

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

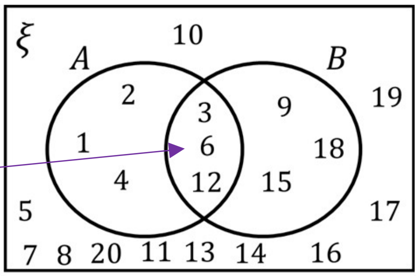
Set A = { Factors of 12} Set A = $\{1, 12, 2, 6, 3, 4\}$

Set B = {Multiples of 3} Set B = $\{3, 6, 9, 12, 15, 18\}$

Intersection
$A \cap B$

What is the set $A \cap B = \{3, 6, 12\}$

What is the set $A \cup B = \{1, 2, 3, 4, 6, 9, 12, 15, 18\}$



Venn diagrams to calculate probabilities

Sami asked 83 people what drink they liked from tea, coffee and cola.

11 people liked tea only
14 people liked coffee only
4 people liked cola only
7 people liked tea and coffee
12 people liked tea and cola
2 people liked tea, coffee and cola.
20 people didn't like any drink

What is the probability that a person chose at random likes coffee and cola but not tea?

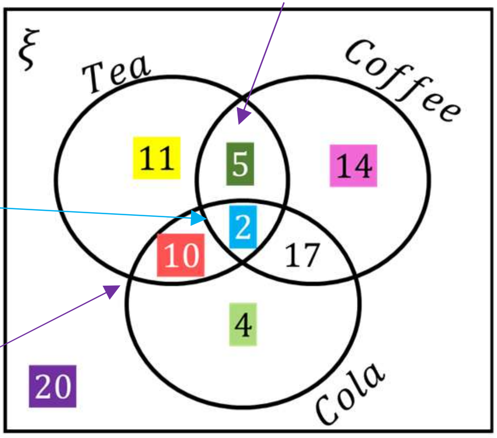
$$P(coffee\ and\ cola\ and\ tea') = \frac{17}{83}$$

Everything (universal set), must add to 83,

Always start with the statement with all 3 mentioned

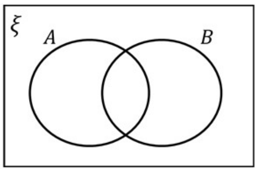
12 people liked tea and cola

7 people liked tea and coffee



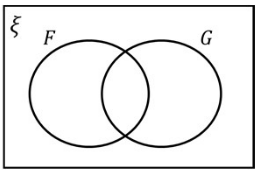
Your turn to practice:

1) ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}
A = multiples of 3
B = multiples of 5
Complete the Venn diagram



One of the numbers is selected at random.
Write down P (A ∩ B)

2) There are 80 students in year 11.
9 students study French and German.
45 students study French
2 students do not study French or German.
Complete the Venn diagram



Work out how many students study only German.

3) In a class of 24 students
12 students play the piano
13 students play the guitar
4 students play neither instrument.

Represent this information on a Venn diagram

A student is selected at random.
Work out the probability that the student only plays the guitar.

Answers
1) $\frac{1}{16}$
2) 33 study German
3) $\frac{8}{24}$

Sample Spaces, Listing & Calculating Probabilities

Keywords and Phrases:

Outcome – possible result of an experiment or trial

Favourable Outcome – outcome that is the required result

Mutually exclusive events - events that cannot occur together

Independent events – the outcome of one event will not effect the outcome of other events

Sample Space Diagram

2-way table showing all possible outcomes from 2 events

The probability of an event happening is written as $P(\text{event})$

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Listing Outcomes and Finding Probabilities

Eg 2 three-sided spinner (red, blue, green)

2-way table

Spinner 2

Spinner 1	Red	Blue	Green
Red	RR	RB	RG
Blue	BR	BB	BG
Green	GR	GB	GG

$\text{P(same colour)} = \frac{3}{9} = \frac{1}{3}$

Structured List

Keep 1 event the same

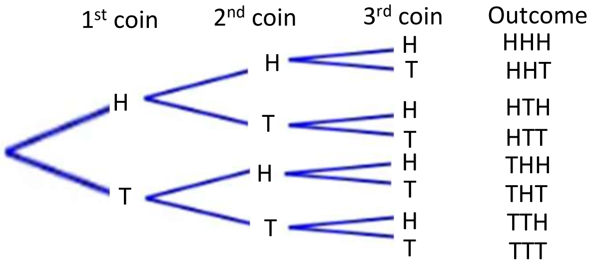
RR, RB, RG
BR, BB, BG
GR, GB, GG

$\text{P(just one green)} = \frac{4}{9}$

All outcomes equally likely

Eg 3 coins

Tree Diagram :useful for several events



$\text{P(exactly 2 heads)} = \frac{3}{8}$

Sample Space Diagram and Finding Probabilities

A sample space diagram is useful when the outcome is not just the list of the events

Eg the sum when 2 fair 6-sided dice are rolled

Dice 2

Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\text{P(the sum is a prime number)} = \frac{14}{36} = \frac{7}{18}$

All outcomes equally likely

Calculating Probabilities of Multiple Events

Spinner 1 has five equal sections, labelled 2, 3, 5, 7 and 11

Spinner 2 has four equal sections, labelled 2, 4, 6 and 7.

The sum of the two scores is found.

Spinner One; Spinner Two

+	2	3	5	7	11
2	4	5	7	9	13
4	6	8	9	11	15
6	8	9	11	13	17
7	9	10	12	14	18

$P(9) = \frac{4}{20} \quad P(11) = \frac{2}{20}$

$P(\text{either } 9 \textbf{ or } 11) = \frac{6}{20}$

$P(\text{either } 9 \textbf{ or } 11) = P(9) + P(11)$

The Or Rule or The Addition Law

$P(A \text{ or } B) = P(A) + P(B)$

True when A and B are mutually exclusive events

Spinner One; Spinner Two

+	2	3	5	7	11
2	4	5	7	9	13
4	6	7	9	11	15
6	8	9	11	13	17
7	9	10	12	14	18

$P(2 \text{ on Spinner one}) = \frac{1}{5}$

$P(4 \text{ on Spinner two}) = \frac{1}{4}$

$P(2 \text{ on Spinner one and } 4 \text{ on Spinner two}) = \frac{1}{20}$

$P(2 \text{ on Spinner one and } 4 \text{ on Spinner two})$
$= P(2 \text{ on Spinner one}) \times \text{P}(4 \text{ on Spinner two})$

The And Rule or The Multiplication Law

$P(A \text{ and } B) = P(A) \times P(B)$

True when A and B are independent events

Practice questions

1. A fair 4-sided spinner is numbered 2, 3, 4 and 5. The spinner is spun twice and the scores added together. Find:
   a)P(total score is 8)
   b)P(total score is less than 7)
2. Joy and Peter both sit a French test. The probability of passing the test is $\frac{2}{5}$. Represent this with a tree diagram. Find the probability that
   (a) they both fail
   (b) only one of them passes
3. The probability that Anna gets up late on a Monday is $\frac{1}{3}$. The probability that she gets up late on a Tuesday is $\frac{1}{4}$. Find the probability that Anna gets up late on one of the days.

Answers: 1.a) $\frac{3}{16}$ b) $\frac{6}{16} = \frac{3}{8}$ 2.a) $\frac{9}{25}$ b) $\frac{12}{25}$ 3. $\frac{5}{12}$

Probability Tree Diagrams

Keywords and Phrases:

Independent events – An Independent Event is an event that is **not affected** by previous events.
Example – Picking something and replacing it, or playing two different games at a fair ground.

Dependant events – Means they **can be affected by previous events.**
Example – Picking something and not replacing it.

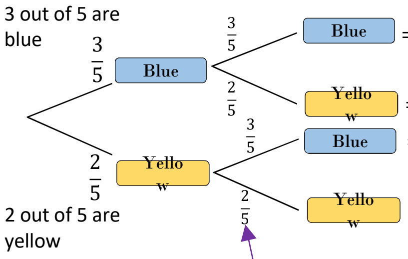
Independent events

Tree diagram for independent event

The outcome of the first event has no bearing on the outcome of the other

Isobel has a bag with 3 blue counters and 2 yellow.
She picks a counter and **replaces it** before the second pick.

$P\ (A \text{ and } B) = P\ (A) \times P\ (B)$



$= P\ (BB)$ $\quad \frac{3}{5}\times\frac{3}{5}=\frac{9}{25}$

$= P\ (BY)$ $\quad \frac{3}{5}\times\frac{2}{5}=\frac{6}{25}$

$= P\ (YB)$ $\quad \frac{2}{5}\times\frac{3}{5}=\frac{6}{25}$

$= P\ (YY)$ $\quad \frac{2}{5}\times\frac{2}{5}=\frac{4}{25}$

Sum of probabilities =1

Because they are replaced the second pick has the same probability

Calculate the probability of picking two different colours

$P(different\ colours) = P(BY)\ or\ P(YB)$

$P\ (A \text{ or } B)\ = P\ (A) + P\ (B)$

$$P(different\ colours) = \frac{6}{25} + \frac{6}{25}$$

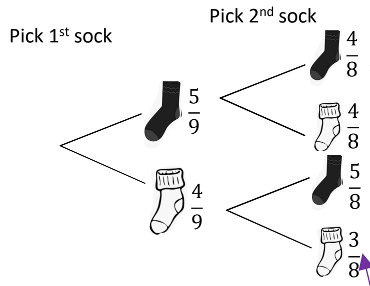
$$= \frac{12}{25}$$

Dependent events

Tree diagram for dependent event

The outcome of the first event has an impact on the second event

A sock drawer has 5 black and 4 white socks.
Jamie picks 2 socks from the drawer and puts them on his feet



$= P\ (BB)$ $\quad \frac{5}{9}\times\frac{4}{8}=\frac{20}{72}$

$= P\ (BW)$ $\quad \frac{5}{9}\times\frac{4}{8}=\frac{20}{72}$

$= P\ (WB)$ $\quad \frac{4}{9}\times\frac{5}{8}=\frac{20}{72}$

$= P\ (WW)$ $\quad \frac{4}{9}\times\frac{3}{8}=\frac{12}{72}$

Sum of probabilities =1

A white sock has been removed

NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced, therefore the denominator is also reduced for the second pick.

Calculate the probability of **at least** one white sock

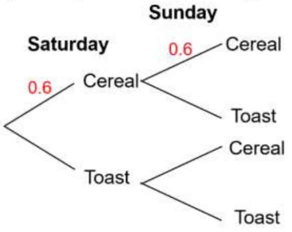
$P(at\ least\ 1\ white)$
$= P(BW)\ or\ P(WB)\ or\ P(WW)$

$P(at\ least\ 1\ white)\quad \frac{20}{72}+\frac{20}{72}+\frac{12}{72} = \frac{52}{72}$

Or $P(at\ least\ 1\ white) = 1 - P(BB$

Your turn to practice:

1) Ray has cereal or toast for his breakfast. The probability that he has cereal is 0.6. Ray has breakfast on Saturday and Sunday. Complete the tree diagram.



Find the probability that Ray has toast for breakfast on both days.

2) Joy and Peter both sit a French test. The probability of passing the test is $\frac{2}{5}$. Represent this with a tree diagram. Find the probability that
(a) only one of them passes the test
(b) they both fail the test

3) The probability that Anna gets up late on a Monday is $\frac{1}{3}$. The probability that she gets up late on a Tuesday is $\frac{1}{4}$. Represent this with a tree diagram. Find the probability that Anna gets up late on at least one of the days.

4) A drawer contains five red socks and three black socks. A sock is taken out at random and not replaced. A second sock is then taken out. Draw a tree diagram and calculate the probability that either a pair of red socks or a pair of black socks is chosen.

5) A bag contains 4 yellow balls and 6 green balls. A ball is taken from the bag and not replaced. A second ball is then taken. Draw a tree diagram and find the probability that the two balls are different colours.

Answers

1) $P(TT) = 0.16$ 2) a) $\frac{12}{25}$ b) $\frac{9}{25}$ 3) $\frac{1}{2}$ 4) $\frac{26}{56} = \frac{13}{28}$ 5) $\frac{48}{90} = \frac{8}{15}$

Product Rule for Counting

Keywords and Phrases:

The product rule for counting is a method for finding the total number of ways of selecting items from a set or sets.

To find the total number of outcomes for two or more events, multiply the number of outcomes for each event together. This is called the product rule for counting because it involves multiplying to find a product.

Product rule for counting when picking from the same set – Order is important:

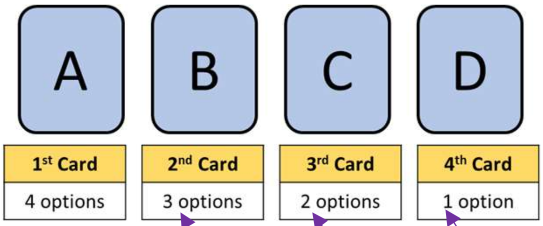
When picking from the same set, if the order is important then we just need to multiply the option available to us on each pick.

Example:

Betsi has 4 cards with the letters A, B, C and D. How many different ways can she arrange these letters?

$$\underbrace{4 \times 3 \times 2 \times 1}_{\text{This is 4 factorial}} = 24$$

4!

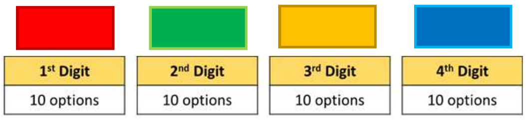


One more option is removed every time

Example

Alex has a 4 digit combination lock. Each digit of the combination can be from 0–9.

How many combinations will there be to his lock?



$$10 \times 10 \times 10 \times 10 = 10\ 000$$

10 000 possible combinations for this code lock.

Product rule for counting when picking from the same set – Order is not important:

When picking from the same set and the order is not important, we must also take into consideration duplicate picks

i.e red, blue will be the same as blue, red.

If we are picking 2 from the same set we must divide by the number of duplicates.

Example:

We must pick 4 girls from a group of 5 to play a doubles tennis match.

How many possible combinations could there be?

Different ways to pick 4 from 5 $= 5 \times 4 \times 3 \times 2$

Duplicate arrangements:

Number of ways 4 girls can be picked

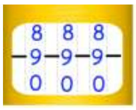
$= 4 \times 3 \times 2 \times 1 = 4!$

$$\frac{\mathit{different\ ways\ to\ pick}}{\mathit{duplicate\ arrangements}} = \frac{5 \times 4 \times 3 \times 2}{4!} = \frac{120}{24} = 5$$

5 **unique** combinations of players

Your turn to practice

1) Shown is a 3-digit combination padlock.



Each dial uses number 0 to 9

a) Work out the total number of different combinations that can be used.

b) Work out the total number of different combinations that have three different digits that can be used.

2) A restaurant has 4 starters and 6 main course on its menu.

Hailey orders a starter and a main course.

How many different combinations of starters and main courses are there?

3) There are 30 students in a class. Two students are going to be selected to receive a prize. How many different pairs of students could be selected?

4) There are 10 teams in a football league. Two teams are going to be chosen at random to play a match.

Work out the number of different matches that could take place.

Answers

1) a) 1000
b) 720
2) 24
3) 435
4) 45