Lytchett Minster School

Year 8
Mathematics
Knowledge organisers

If you lose your Knowledge organiser you will be asked to replace it at a cost of 50p per copy.

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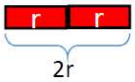
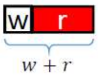
2024/2025

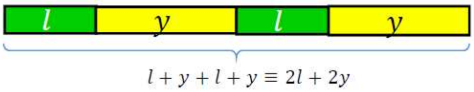
Forming algebraic expressions

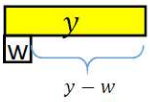
Expressions

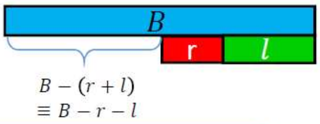
To add expressions we use a part/whole model

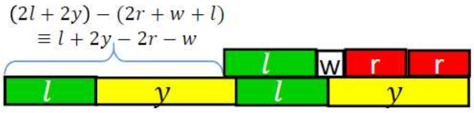




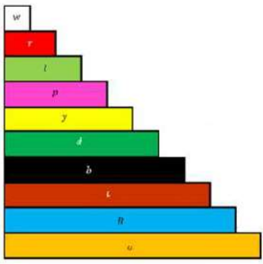








Always simplify the expressions

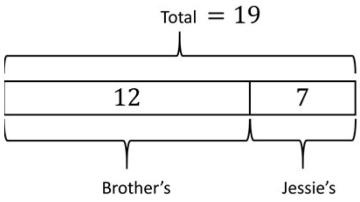


Brackets allow you to group different variables together

Part-part whole bar model

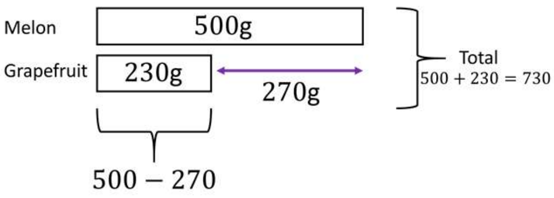
Jessie gives her brother 12 chocolates. She has 7 chocolates left.
Draw a bar model to represent this situation.

How many chocolates did Jessie have to start with?



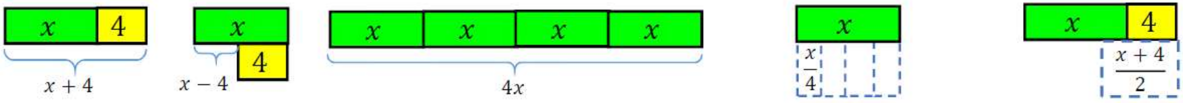
Comparison bar model

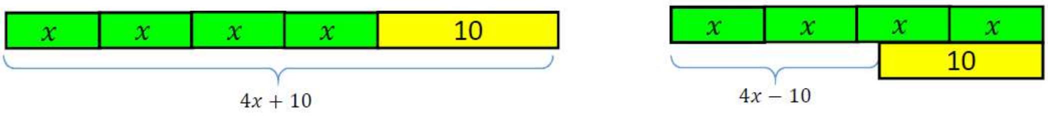
A melon weighs 500g and a grapefruit weighs 270g less.
Draw a bar model to represent this.
How much does the grapefruit weigh?
What is the total weight?



Forming algebraic expressions using bar models:

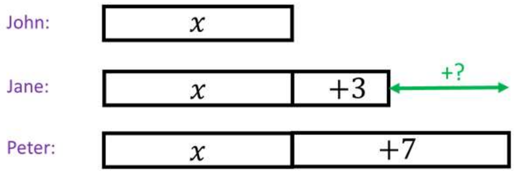
Using part-part whole bar models:



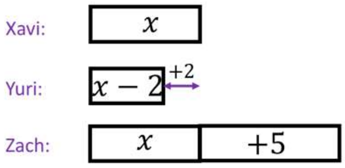


Using Comparison bar models:

John is x years old.
Jane is 3 years older than John.
Peter is seven years older than John.
How much older is Peter than Jane



Xavi has some money.
Yuri has £2 less than Xavi.
Zach has £5 more than Xavi.
Can you find an expression for how much money they have in total?



$$x + x - 2 + x + 5$$

$$3x + 3$$

Your turn to practice:

1) James has 15 sweets. Lucy has 7 fewer sweets than James. Max has twice as many sweets as Lucy. Draw a bar model to represent this.
How many sweets does Lucy have?
How many sweets do they have altogether?

2) A child ticket to the cinema costs £c. An adult ticket is twice as much. Find an expression for the total cost of 3 child and 2 adult tickets

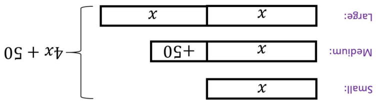
3) A small parcel weighs x grams. A medium parcel weighs 50g more than the small parcel. A large parcel weighs twice as much as the small parcel. Draw a bar model to represent the weights of the 3 parcels. Find an expression for the total weight of the 3 parcels

Answers

1) Lucy has 8 sweets. There are 39 sweets altogether

2) An adult ticket = 2c
$3c + 2(2c) = 3c + 4c =$ **$7c$**

3)



Linear equations – Bar Models

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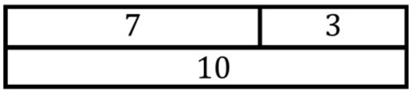
Equivalence bar model



This is an equivalence bar model and represents the calculation:

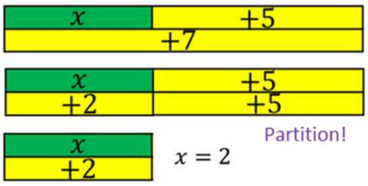
$$7 + 3 = 10$$

It could also represent:

$3 + 7 = 10$ $\quad$ $10 - 3 = 7$ $\quad$ $10 - 7 = 3$

Solving equations - Addition

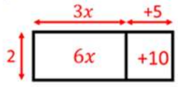
Solve: $x + 5 = 7$



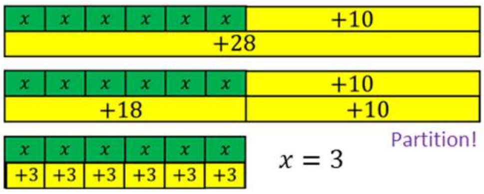
$x = 2$

Solve: $2(3x + 5) = 28$

Expand bracket first

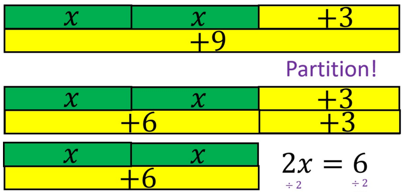


$6x + 10 = 28$



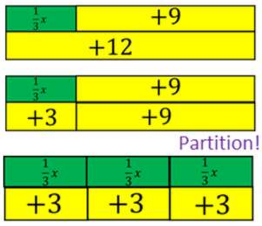
$x = 3$

Solve: $2x + 3 = 9$



$$\begin{aligned} 2x &= 6 \\ \div 2 \quad & \quad \div 2 \\ x &= 3 \end{aligned}$$

Solve: $\frac{x}{3} + 9 = 12$



$x = 9$

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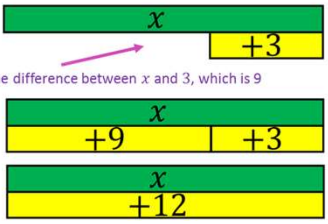
Solving equations - Subtraction

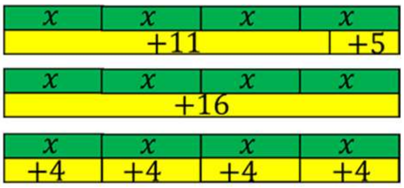
Solve: $x - 3 = 9$



This gap represents the difference between x and 3, which is 9

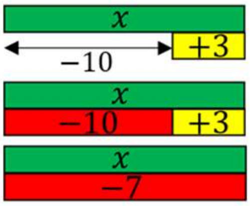
$x = 12$

Solve: $4x - 5 = 11$



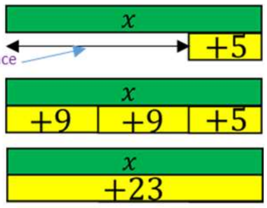
$x = 4$

Solve: $x - 3 = -10$



$x = -7$

Solve: $\frac{x-5}{2} = 9$



This difference is 9 × 2

$x = 23$

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Your turn to practice:

Solve

1) $y + 7 = 15$
2) $7 + t = 14$
3) $13 = x + 4$
4) $41 = x + 23$
5) $k + 2 = -4$
6) $7x + 4 = 39$
7) $28 = 8x + 4$
8) $2x + 3 = -29$
9) $3t + 12 = 9$
10) $3x + 41 = 5$
11) $2(x + 1) = 18$
12) $3(2x + 1) = 9$
13) $5(2 + 3x) = 25$
14) $\frac{1}{2}(6x + 8) = 19$
15) $7(3x + 8) = 14$

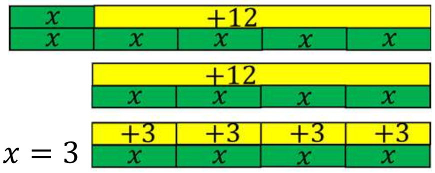
Answers

1) $y = 8$
2) $t = 7$
3) $x = 9$
4) $x = 18$
5) $k = -6$
6) $x = 5$
7) $x = 3$
8) $x = -16$
9) $t = -1$
10) $x = -12$
11) $x = 8$
12) $x = 1$
13) $x = 1$
14) $x = 5$
15) $x = -2$

Linear equations – Bar Models 2

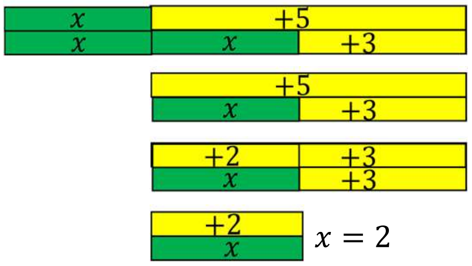
Solving equations, unknowns both sides - Addition

Solve: $x + 12 = 5x$



$x = 3$

Solve: $x + 5 = 2x + 3$

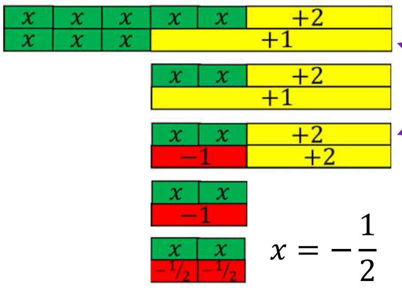


$x = 2$

Solve: $5x + 2 = 1 + 3x$

$5x + 2 = 3x + 1$

We have used the commutative law as we want the unknowns to line up in our bar model.



The magnitude of +2 is greater than +1, so there will be negatives to consider here.

Partition!

$x = -\frac{1}{2}$

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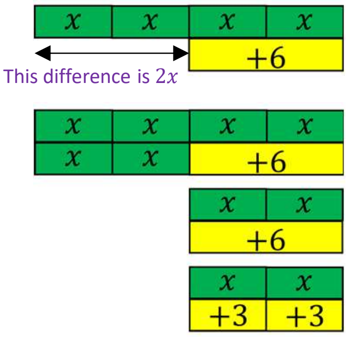
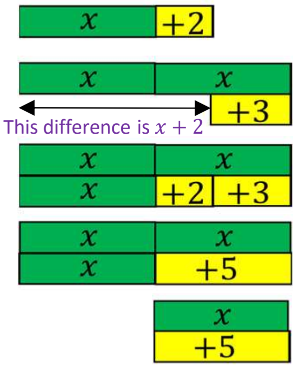
Solving equations, unknowns both sides - Subtraction

Solve: $4x - 6 = 2x$



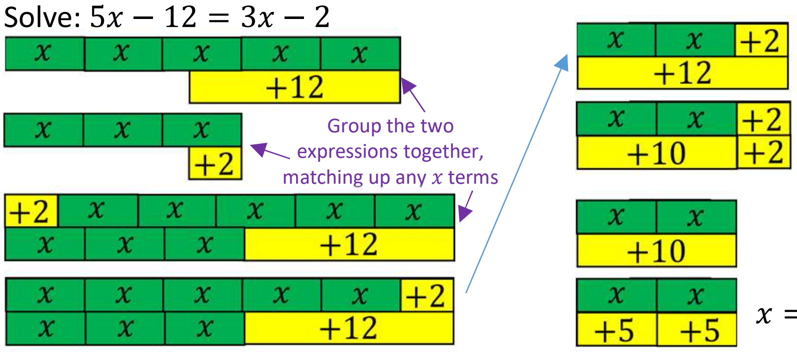
$x = 3$

Solve: $x + 2 = 2x - 3$



$x = 5$

Solve: $5x - 12 = 3x - 2$



Group the two expressions together, matching up any x terms

Partition!

$x = 5$

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Your turn to practice: Solve, using bar models

1) $5a = 21 + 2a$
2) $b + 15 = 6b$
3) $2c + 20 = 4c$
4) $4d + 3 = 2d + 11$
5) $5e + 7 = 3e + 11$
6) $2f + 21 = 8f + 3$
7) $4g - 5 = 3g$
8) $7h + 7 = 9h - 1$
9) $6j - 2 = 5j + 4$
10) $12k - 4 = 7k - 29$
11) $4m - 7 = m - 1$
12) $8n - 7 = 6n - 4$

Answers
1) $a = 7$
2) $b = 3$
3) $c = 10$
4) $d = 4$
5) $e = 2$
6) $f = 3$
7) $g = 5$
8) $h = 4$
9) $j = 6$
10) $k = -5$
11) $m = 2$
12) $n = 1.5$

Adding and Subtracting fractions

Keywords and Phrases:

Unit fraction - One equal part of a whole is called a unit fraction

Common multiples – Multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18,
Multiples of 3 are 3, 6, 9, 12, 15, 18

There are infinite common multiples of numbers but only one lowest common multiple.

So, the lowest common multiple of 2 and 3 is 6. **LCM (2,3) = 6**

Adding and subtracting same denominator:

When adding or subtracting fractions with the same denominator we ONLY add or subtract the numerators.

$$\frac{a}{m}+\frac{b}{m}=a\times\frac{1}{m}+b\times\frac{1}{m}$$

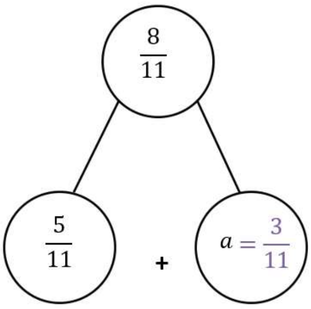
$$=(a+b)\times\frac{1}{m}$$

Multiple of a Unit fraction

$$\frac{a}{m}-\frac{b}{m}=a\times\frac{1}{m}-b\times\frac{1}{m}$$

$$=(a-b)\times\frac{1}{m}$$

Example: Pictorial Representation



$$\frac{5}{11}+a=\frac{8}{11}$$

Formal Working

$$a=\frac{8}{11}-\frac{5}{11}$$

$$a=8\times\frac{1}{11}-5\times\frac{1}{11}$$

$$a=(8-5)\times\frac{1}{11}$$

$$a=3\times\frac{1}{11}$$

Adding and subtracting different denominators:

To add or subtract fractions we need to be adding multiples of the same unit.
This means that we need fractions with the same denominator.

$$\frac{1}{12}+\frac{1}{4}$$

Pictorial Method

$$\frac{1}{12}+\frac{3}{12}=\frac{4}{12}$$

Lowest Common Multiple

Formal Method

$$\frac{1}{12}+\frac{1}{4}\xrightarrow{\times 3}\frac{1}{12}+\frac{3}{12}=\frac{4}{12}$$

Multiples of 4 - 4, 8, 12
Multiples of 12 - 12, 24, 36

Lowest Common Multiple

Your turn to practice:

1) $\frac{1}{3}+\frac{2}{3}=$
2) $\frac{2}{5}+\frac{1}{5}=$
3) $\frac{7}{6}+\frac{3}{6}=$
4) $\frac{3}{15}-\frac{1}{15}=$
5) $\frac{5}{13}-\frac{2}{13}=$
6) $\frac{1}{3}+\frac{1}{6}=$
7) $\frac{2}{10}+\frac{3}{5}=$
8) $\frac{1}{6}+\frac{3}{18}=$
9) $\frac{7}{15}-\frac{1}{5}=$
10) $\frac{9}{14}-\frac{2}{7}=$
11) $\frac{1}{5}+\frac{1}{6}=$
12) $\frac{2}{4}+\frac{3}{5}=$
13) $\frac{1}{6}+\frac{3}{7}=$
14) $\frac{6}{7}-\frac{1}{5}=$
15) $\frac{9}{12}-\frac{2}{7}=$

Answers

1) $\frac{3}{3}=1$
2) $\frac{3}{5}$
3) $\frac{10}{6}$
4) $\frac{2}{15}$
5) $\frac{3}{13}$
6) $\frac{1}{2}$
7) $\frac{4}{5}$
8) $\frac{1}{3}$
9) $\frac{4}{15}$
10) $\frac{5}{14}$
11) $\frac{11}{30}$
12) $\frac{11}{10}$
13) $\frac{25}{42}$
14) $\frac{23}{35}$
15) $\frac{13}{28}$

+ or − Mixed number fractions

Keywords and Phrases:

Mixed number – A mixed number is a whole number and a proper fraction combined

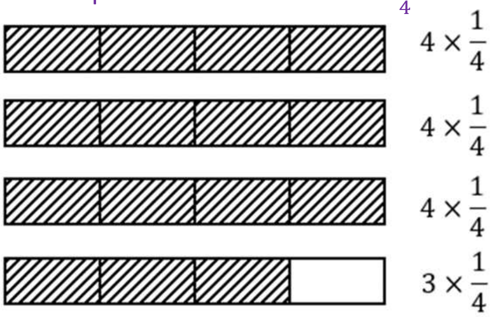
Proper fraction - A fraction where the numerator is less than the denominator.

Improper fraction – An Improper Fraction has a numerator larger than (or equal to) the denominator.

Writing improper fractions as mixed numbers

$$\frac{15}{4} = 15 \times \frac{1}{4}$$
$$= \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$
$$= 1 + 1 + 1 + \frac{3}{4}$$
$$= 3\frac{3}{4}$$

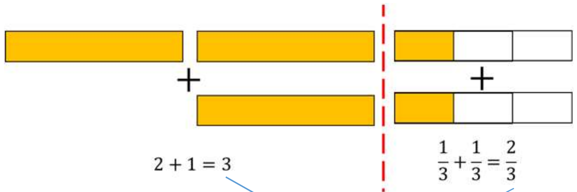
Pictorial representation of 15 lots of $\frac{1}{4}$



Adding and subtracting mixed numbers same denominator

Using **partitioning** to add mixed numbers:

$$2\frac{1}{3} + 1\frac{1}{3}$$



$$3\frac{2}{3}$$

Or….. Convert into improper fractions to add

$$2\frac{1}{3} = \frac{7}{3} \qquad 1\frac{1}{3} = \frac{4}{3}$$
$$\frac{7}{3} + \frac{4}{3} = \frac{11}{3}$$

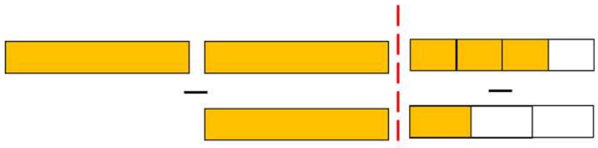
Convert back into a mixed number

$$\frac{11}{3} = 3\frac{2}{3}$$

Adding and subtracting mixed numbers different denominator

Using **partitioning** to subtract mixed numbers:

$$2\frac{3}{4} - 1\frac{1}{3}$$



$$2 - 1 = 1$$
$$\frac{3}{4} - \frac{1}{3}$$
$$\frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

Common denominator

$$1\frac{5}{12}$$

Or….. Convert into improper fractions to subtract

$$2\frac{3}{4} = \frac{11}{4} \qquad 1\frac{1}{3} = \frac{4}{3}$$

Common denominator

$$\frac{11}{4} - \frac{4}{3} = \frac{33}{12} - \frac{16}{12} = \frac{17}{12}$$

Convert back into a mixed number

$$\frac{17}{12} = 1\frac{5}{12}$$

Your turn to practice:

Convert into mixed numbers

1) $\frac{11}{4} =$

2) $\frac{27}{7} =$

Convert into Improper fractions

3) $3\frac{4}{5} =$

4) $4\frac{5}{7} =$

5) $5\frac{7}{8}$

Calculate, partitioning method is easier and simplify:

6) $1\frac{1}{4} + 2\frac{1}{4} =$

7) $1\frac{1}{5} + 3\frac{2}{5} =$

8) $1\frac{4}{5} + 2\frac{3}{5} =$

9) $4\frac{3}{7} - 1\frac{2}{7} =$

10) $5\frac{5}{8} - 2\frac{3}{8} =$

11) $1\frac{1}{3} + 2\frac{3}{4} =$

12) $1\frac{1}{5} + 3\frac{2}{7} =$

13) $1\frac{4}{5} + 2\frac{3}{8} =$

14) $4\frac{1}{5} - 1\frac{2}{7} =$

15) $5\frac{1}{5} - 2\frac{3}{8} =$

Answers

1) $2\frac{3}{4}$
2) $3\frac{6}{7}$
3) $\frac{19}{5}$
4) $\frac{33}{7}$
5) $\frac{47}{8}$
6) $3\frac{1}{2}$
7) $4\frac{3}{5}$
8) $4\frac{2}{5}$
9) $3\frac{1}{7}$
10) $3\frac{1}{4}$
11) $4\frac{1}{12}$
12) $4\frac{17}{35}$
13) $4\frac{7}{40}$
14) $2\frac{32}{35}$
15) $2\frac{33}{40}$

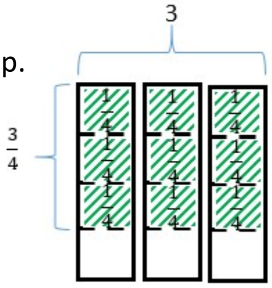
Multiply and Simplify Fractions

Multiply an integer by a fraction:

To multiply a fraction by an integer draw an array to help.

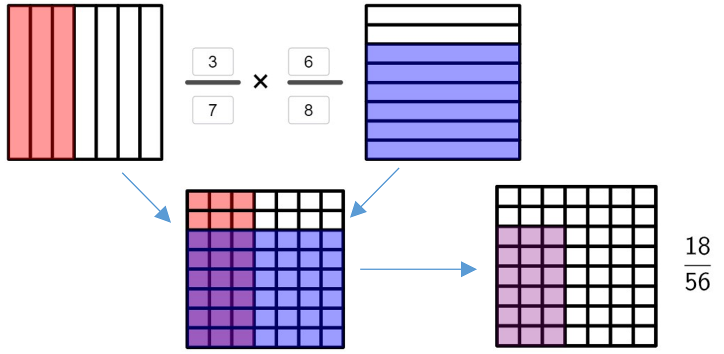
Example: $\frac{3}{4} \times 3 = \frac{9}{4}$

$$\boxed{c \times \frac{a}{b} = \frac{a \times c}{b}} \quad \boxed{\frac{a}{b} \times c = \frac{a \times c}{b}}$$



Multiply a fraction by a fraction:

Use an array to help:



Alternatively you could calculate like this:

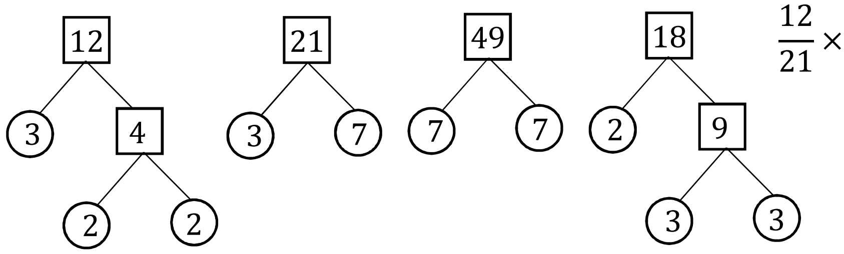
$$\frac{3}{7} \times \frac{6}{8} = \frac{3 \times 6}{7 \times 8}$$

$$= \frac{18}{56}$$

$$\boxed{\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}}$$

Sometimes it is best to simplify using product of primes before you multiply, e.g:

$\frac{12}{21} \times \frac{49}{18}$



$12 = 3 \times 2 \times 2$ $21 = 3 \times 7$ $49 = 7 \times 7$ $18 = 2 \times 3 \times 3$

$$\frac{12}{21} \times \frac{49}{18} = \frac{3 \times 2 \times 2 \times 7 \times 7}{3 \times 7 \times 2 \times 3 \times 3}$$

$$= \frac{2 \times 2 \times 3 \times 7 \times 7}{2 \times 3 \times 3 \times 3 \times 7}$$

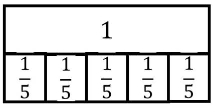
$$= \frac{2}{2} \times \frac{3}{3} \times \frac{7}{7} \times \frac{2 \times 7}{3 \times 3}$$

$$= 1 \times 1 \times 1 \times \frac{14}{9} = 1\frac{5}{9}$$

Reciprocals:

The reciprocal is the number that we have to multiply by to make one.

$5 \times \frac{1}{5} = 1$ Five lots of one fifth.



The reciprocal of 5 is $\frac{1}{5}$

$\frac{1}{5} \times 5 = 1$ ← The commutative law for multiplication!

The reciprocal of $\frac{1}{5}$ is 5

$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{2}{2} \times \frac{3}{3} = 1 \times 1 = 1$

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

Your turn to practice:

Calculate and simplify:

1) $\frac{1}{3} \times 5 =$
2) $3 \times \frac{1}{7} =$
3) $\frac{2}{5} \times 4 =$
4) $4 \times \frac{3}{16} =$
5) $-\frac{6}{3} \times -4 =$
6) $\frac{1}{3} \times \frac{1}{5} =$
7) $\frac{3}{5} \times \frac{1}{7} =$
8) $\frac{2}{5} \times \frac{3}{2} =$
9) $\frac{4}{15} \times \frac{3}{16} =$
10) $-\frac{16}{13} \times -\frac{13}{4} =$
11) What is the reciprocal of 7
12) What is the reciprocal of $\frac{1}{6}$
13) What is the reciprocal of $\frac{5}{9}$

Answers:

1) $\frac{5}{3}$
2) $\frac{3}{7}$
3) $\frac{8}{5} = 1\frac{3}{5}$
4) $\frac{3}{4}$
5) 8
6) $\frac{1}{15}$
7) $\frac{3}{35}$
8) $\frac{3}{5}$
9) $\frac{1}{20}$
10) 4
11) $\frac{1}{7}$
12) 6
13) $\frac{9}{5}$

Fractions Division

Keywords and Phrases:

$5 \times \frac{1}{5} = 1$ The reciprocal of 5 is $\frac{1}{5}$

Reciprocal - The reciprocal is the number that we have to multiply by to make one.

Unitisation - a mathematical term used to describe counting groups of the same number of things as single units. E.g $\frac{4}{5}$ is $4\ lots\ of\ \frac{1}{5}$ or $4 \times \frac{1}{5}$

Dividing a fraction by an integer:

Dividing by an integer is equivalent to multiplying by its reciprocal.

Example:

$$\frac{1}{5} \div 2 = \frac{1}{5} \times \frac{1}{2}$$

$$= \frac{1}{10}$$

Example:

$$\frac{2}{3} \div 6 = \frac{2}{3} \times \frac{1}{6} = \frac{2 \times 1}{3 \times 2 \times 3}$$

Don't forget to simplify before you multiply

$$= \frac{2}{2} \times \frac{1}{3 \times 3} = 1 \times \frac{1}{9} = \frac{1}{9}$$

$$\boxed{\frac{1}{a} \div b = \frac{1}{a} \times \frac{1}{b}}$$

Dividing an integer by a fraction:

Dividing an integer by a fraction is equivalent to multiplying by its reciprocal.

Example:

$$4 \div \frac{3}{5} = 4 \times \frac{5}{3}$$

$$= \frac{4 \times 5}{3} = \frac{20}{3} = 6\frac{2}{3}$$

$$\boxed{a \div \frac{b}{c} \equiv a \times \frac{c}{b}}$$

Dividing fractions with the same denominator:

Using unitisation method works really well here.

Example:

$$\frac{4}{5} \div \frac{2}{5} = 4 \times \frac{1}{5} \div 2 \times \frac{1}{5}$$

$$4 \div 2 = 2$$

Dividing fractions with different denominator:

Dividing a fraction by a fraction is equivalent to multiplying by it's reciprocal.

What is the reciprocal of $\frac{2}{3}$?

$$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$$

$$= \frac{12}{10} = 1\frac{1}{5}$$

Your turn to practice:

Calculate and simplify:

1) $\frac{1}{3} \div 5 =$
2) $3 \div \frac{1}{7} =$
3) $\frac{2}{5} \div 4 =$
4) $16 \div \frac{4}{3} =$
5) $-\frac{6}{3} \div -4 =$
6) $\frac{1}{5} \div \frac{2}{5} =$
7) $\frac{6}{7} \div \frac{3}{7} =$
8) $\frac{4}{15} \div \frac{2}{15} =$
9) $\frac{16}{15} \div \frac{4}{15} =$
10) $-\frac{18}{13} \div -\frac{9}{13} =$
11) $\frac{1}{5} \div \frac{1}{6} =$
12) $\frac{6}{5} \div \frac{3}{7} =$
13) $\frac{4}{15} \div \frac{7}{3} =$
14) $\frac{16}{8} \div \frac{4}{24} =$
15) $-\frac{6x}{5} \div -\frac{x}{20} =$

Answers:
1) $\frac{1}{15}$
2) 21
3) $\frac{1}{10}$
4) 12
5) $\frac{1}{2}$
6) $\frac{1}{2}$
7) 2
8) 2
9) 4
10) 2
11) $\frac{6}{5} = 1\frac{1}{6}$
12) $\frac{14}{5} = 2\frac{4}{5}$
13) $\frac{4}{35}$
14) 12
15) 24

× or ÷ Mixed number fractions

Multiplying mixed number fractions:

To multiply mixed number fractions, convert to improper fractions first then multiply numerators and multiply denominators.

Example: $2\frac{3}{4} \times 1\frac{2}{3}$

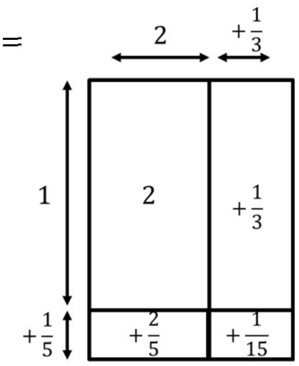
Convert each number to improper fraction first

$$2\frac{3}{4} = \frac{11}{4} \qquad 1\frac{2}{3} = \frac{5}{3} \longrightarrow \frac{11}{4} \times \frac{5}{3} \quad = \frac{11 \times 5}{4 \times 3} \quad = \frac{55}{12} \quad = 4\frac{7}{12}$$

Alternatively, use an **area model** to calculate

This method is very similar to how you would calculator 42 × 65

Example: $1\frac{1}{5} \times 2\frac{1}{3} =$



$$= 2 + \frac{1}{3} + \frac{2}{5} + \frac{1}{15}$$

Add up all of the areas

$$= 2 + \frac{5}{15} + \frac{6}{15} + \frac{1}{15}$$

Common denominator to add

$$= 2 + \frac{12}{15}$$

$$= 2 + \frac{4}{5} \qquad = 2\frac{4}{5}$$

Simplify, and your answers is given as a mixed number

Dividing mixed number fractions:

To divide mixed number fractions, convert to improper fractions first then divide by multiplying by the reciprocal or unitisation method.

Example: $6\frac{3}{8} \div 2\frac{1}{2} =$

Convert each number to improper fraction first

Write this as an equivalent calculation using multiplication

$$6\frac{3}{8} = \frac{51}{8} \qquad 2\frac{1}{2} = \frac{5}{2} \longrightarrow 6\frac{3}{8} \div 2\frac{1}{2} = \frac{51}{8} \div \frac{5}{2} \longrightarrow \frac{51}{8} \times \frac{2}{5}$$

What is the reciprocal of $\frac{5}{2}$?

$$= \frac{51}{2 \times 4} \times \frac{2}{5}$$

$$= \frac{2}{2} \times \frac{51}{4 \times 5}$$

$$= 1 \times \frac{51}{20}$$

$$= 2\frac{11}{20}$$

Convert back into a mixed number

Your turn to practice:

Calculate and simplify:

1) $2\frac{1}{2} \times 1\frac{1}{2}$

2) $1\frac{1}{4} \times 3\frac{1}{2}$

3) $2\frac{1}{3} \times 3\frac{1}{6}$

4) $2\frac{5}{6} \times 2\frac{2}{3}$

5) $2\frac{3}{7} \times 1\frac{2}{5}$

6) $\frac{2}{3} \div 2\frac{1}{5}$

7) $1\frac{2}{3} \div 2\frac{1}{5}$

8) $2\frac{2}{3} \div 2\frac{1}{5}$

9) $2\frac{2}{3} \div 1\frac{1}{5}$

10) $1\frac{2}{5} \div 2\frac{1}{3}$

Answers

1) $3\frac{3}{4}$ 6) $\frac{10}{33}$

2) $4\frac{3}{8}$ 7) $\frac{25}{33}$

3) $7\frac{7}{18}$ 8) $1\frac{7}{33}$

4) $7\frac{5}{9}$ 9) $2\frac{2}{9}$

5) $3\frac{2}{5}$ 10) $\frac{3}{5}$

Linking numbers with Multiplication

Keywords and Phrases:

Reciprocal -The reciprocal is the number that we have to multiply by to make one.

$\frac{1}{2} \times \boxed{2} = 1$ One half of two is a whole.

$\boxed{5} \times \frac{1}{5} = 1$ Five lots of one fifth is a whole.

$\boxed{\frac{1}{10}} \times 10 = 1$ One tenth of ten is a whole.

$\boxed{\frac{1}{a} \times a = 1 \qquad a \neq 0 \qquad a \text{ is the reciprocal of } \frac{1}{a}}$

Linking numbers by multiplying

What is the missing number?

$13 \times \boxed{} = 22$

What is the reciprocal of 13? $= \frac{1}{13}$

$\underbrace{13 \times \frac{1}{13}}_{=1} \times 22 = 22$

$13 \times \boxed{\frac{22}{13}} = 22$

$\frac{2}{3} \times \boxed{} = 5$

What is the reciprocal of $\frac{2}{3}$? $= \frac{3}{2}$

$\underbrace{\frac{2}{3} \times \frac{3}{2}}_{=1} \times 5 = 5$

$\frac{2}{3} \times \boxed{\frac{15}{2}} = 5$

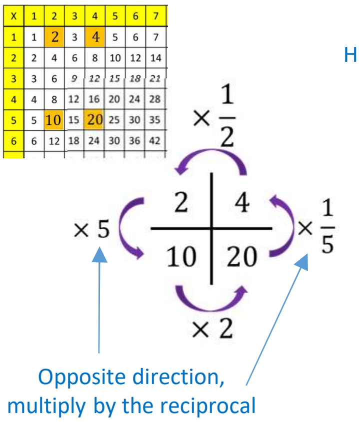
Any two numbers can be linked by a multiplication.

$\boxed{a \times \frac{b}{a} = b \qquad a \neq 0}$

$\boxed{\frac{a}{b} \times \frac{b \times c}{a} = c \qquad a, b \neq 0}$

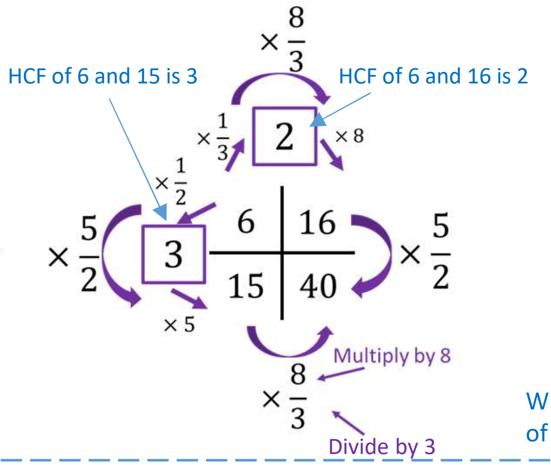
Proportional reasoning grids

Any set of four numbers from the times table grid have similar multiplicative relationships.



x	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	8	10	12	14
3	3	6	9	12	15	18	21
4	4	8	12	16	20	24	28
5	5	10	15	20	25	30	35
6	6	12	18	24	30	36	42

Opposite direction, multiply by the reciprocal

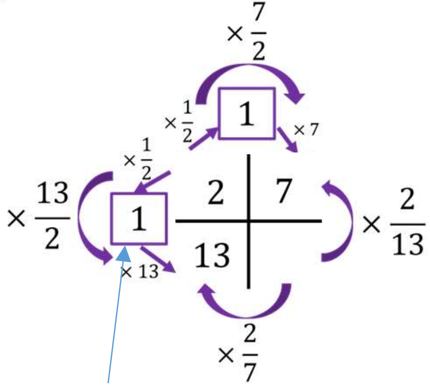


HCF of 6 and 15 is 3

HCF of 6 and 16 is 2

Multiply by 8

Divide by 3



When two numbers have a HCF of 1, they are called co-prime

Your turn to practice:

Fill in the blanks to complete the calculations

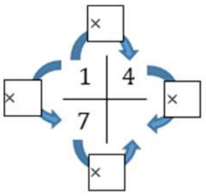
1) $4 \times \boxed{} = 1$

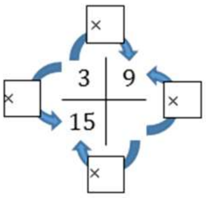
2) $\frac{2}{3} \times \boxed{} = 1$

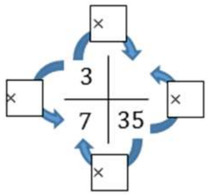
3) $9 \times \boxed{} = 15$

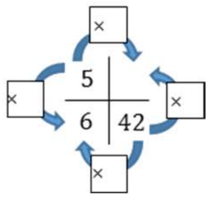
4) $\frac{2}{5} \times \boxed{} = 3$

Fill in the blanks to fully describe the multiplicative relationships









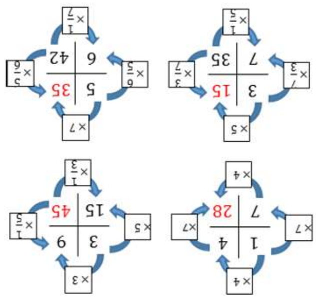
Answers

1) $4 \times \frac{1}{4} = 1$

3) $9 \times \frac{15}{9} = 15$

2) $\frac{2}{3} \times \frac{3}{2} = 1$

4) $\frac{2}{5} \times \frac{15}{2} = 3$



Linear equations - Formal Working

Solving equations formal working – Addition

You must **always partition** then use **zero pairs**.

Solve: $x + 5 = 7$

Partition 7 into $2 + 5$

$$\begin{array}{rcl} x + 5_0 &=& 7 \\ x + \overset{0}{\cancel{5}} &=& 2 + \overset{0}{\cancel{5}} \\ \cancel{-5} & \Big| & \cancel{-5} \\ x &=& 2 \end{array}$$

Solve: $2x + 3 = 9$

Partition 9 into $6 + 3$

$$\begin{array}{rcl} 2x + 3 &=& 9 \\ 2x + \overset{0}{\cancel{3}} &=& 6 + \overset{0}{\cancel{3}} \\ \cancel{-3} & \Big| & \cancel{-3} \\ 2x &=& 6 \\ \times\frac{1}{2} & \Big| & \times\frac{1}{2} \\ x &=& 3 \end{array}$$

Multiply by the reciprocal of 2

Solve: $5x + 2 = 2x + 8$

Partition $5x$ into $2x + 3x$

$$\begin{array}{rcl} 5x + 2 &=& 2x + 8 \\ \overset{0}{\cancel{2x}} + 3x + 2 &=& \overset{0}{\cancel{2x}} + 8 \\ \cancel{-2x} & \Big| & \cancel{-2x} \\ 3x + 2 &=& 8 \\ 3x + \overset{0}{\cancel{2}} &=& 6 + \overset{0}{\cancel{2}} \\ \cancel{-2} & \Big| & \cancel{-2} \\ 3x &=& 6 \\ \div\frac{1}{3} & \Big| & \times\frac{1}{3} \\ x &=& 2 \end{array}$$

Partition 8 into $6 + 2$

Multiply by the reciprocal of 3

Key Points:

- Partition before zero pairs
- Always show zero pairs
- To isolate the x multiply by the reciprocal

If there is an unknown on both sides, make a zero-pair with your x (or whatever your unknown is) first, and then solve as normal…

Solving equations formal working – Subtraction

You **don't need to partition**, just show **zero pairs**

Solve: $x - 3 = 9$

$$\begin{array}{rcl} x - \overset{0}{\cancel{3}} &=& 9 \\ \cancel{+3} & \Big| & +3 \\ x &=& 12 \end{array}$$

Solve: $6x - 4 = -28$

$$\begin{array}{rcl} 6x - \overset{0}{\cancel{4}} &=& -28 \\ \cancel{+4} & \Big| & +4 \\ 6x &=& -24 \\ \times\frac{1}{6} & \Big| & \times\frac{1}{6} \\ x &=& -4 \end{array}$$

Remember to make zero pairs, and then multiply by the reciprocal to isolate the x…

Solve: $6x - 9 = 10x - 25$

$$\begin{array}{rcl} \overset{0}{\cancel{6x}} - 9 &=& \overset{0}{\cancel{6x}} + 4x - 25 \\ \cancel{-6x} & \Big| & \cancel{-6x} \\ -9 &=& 4x - \overset{0}{\cancel{25}} \\ +25 & \Big| & \cancel{+25} \\ 16 &=& 4x \\ \times\frac{1}{4} & \Big| & \times\frac{1}{4} \\ 4 &=& x \\ x &=& 4 \end{array}$$

Multiply by the reciprocal of 4

Key Points:

- No need to partition when subtracting.
- Always show zero pairs
- To isolate the x multiply by the reciprocal

Your turn to practice:

Solving equations formal working – Addition

1. $x + 5 = 17$
2. $15 + x = 24$
3. $x + 7 = 12$
4. $x + 8 = 19$
5. $4 + x = 21$
6. $2x + 3 = 13$
7. $4x + 1 = 17$
8. $9 + 3x = 27$
9. $18 = 5x + 3$
10. $2x + 1 = 5$

Solving equations formal working – Subtraction

1. $x - 3 = 7$
2. $x - 11 = 15$
3. $5 - x = 2$
4. $x - 8 = 20$
5. $13 = x - 4$
6. $2x - 3 = 5$
7. $4a - 5 = 11$
8. $7m - 3 = 18$
9. $6k - 9 = 15$
10. $3x - 2 = 13$

Challenge:

1. $3(2d + 1) = 9$
2. $2(5y - 3) = 14$
3. $15 = \frac{b}{2} + 6$
4. $\frac{x+6}{2} = 10$
5. $7 + 5y = 3y - 5$
6. $4y - 3 = 3y + 1$
7. $5(x - 4) = 3x - 7$
8. $2(3p - 3) = 5(p - 1)$

Answers
Addition

1.	$x = 12$	6.	$x = 5$
2.	$x = 9$	7.	$x = 4$
3.	$x = 5$	8.	$x = 6$
4.	$x = 11$	9.	$x = 3$
5.	$x = 17$	10.	$x = 2$

Answers
Subtraction

1.	$x = 10$	6.	$x = 4$
2.	$x = 26$	7.	$x = 4$
3.	$x = 3$	8.	$x = 3$
4.	$x = 28$	9.	$x = 4$
5.	$x = 17$	10.	$x = 5$

Challenge

1.	$d = 1$	5.	$y = -6$
2.	$y = 2$	6.	$y = 4$
3.	$b = 18$	7.	$x = 6.5$
4.	$x = 14$	8.	$p = 1$

Proportion

Keywords and Phrases:

Proportion - a part, a share, or number considered in comparative relation to a whole.

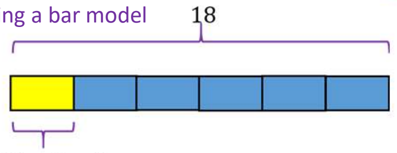
Percent – The parts per 100, a ratio “out of 100”

Fractions of an amount:

Example 1: Find $\frac{1}{6}$ of 18

$\frac{1}{6}$ 1 of the equal parts; 6 equal parts

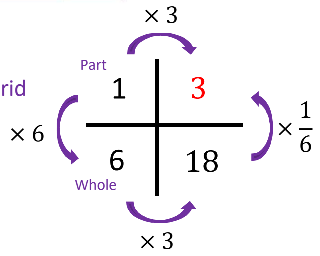
Using a bar model



$18 \div 6 = 3$

$18 \times \frac{1}{6} = 3$

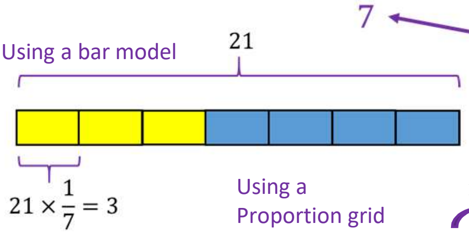
Using a Proportion grid



Example 2: Find $\frac{3}{7}$ of 21

$\frac{3}{7}$ Multiply by 3; Divide by 7

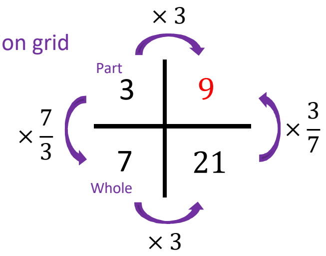
Using a bar model



$21 \times \frac{1}{7} = 3$

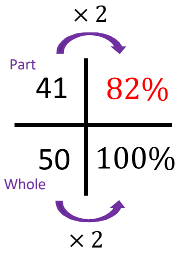
$3 \times 3 = 9$

Using a Proportion grid



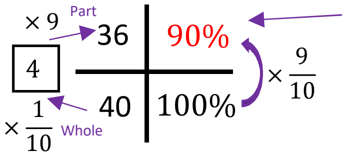
Writing a Number as a Percentage of a Quantity – Non Calc

Example 1: Bob scored 41 out of 50 on his test.
What is this as a percentage?



$$\frac{41}{50} = 82\%$$

Example 2: Jane scored 36 out of 40 on her test.
What is this as a percentage?

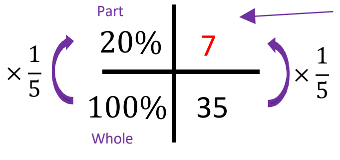


$100 \times \frac{9}{10} = 90$

$$\frac{36}{40} = 90\%$$

Percentages of an amount

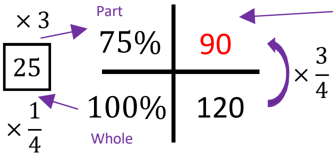
Example 1: Find 20% of 35



$35 \times \frac{1}{5} = 7$

20% of 35 = 7

Example 2: Find 75% of 120



$120 \times \frac{3}{4} = 90$

75% of 120 = 90

Your turn to practice

Write as a percentage:

1) Find $\frac{2}{3}$ of 9.
2) Find $\frac{3}{5}$ of 15.
3) Find $\frac{3}{4}$ of 8.
4) Find $\frac{6}{11}$ of 55.
5) Find $\frac{4}{5}$ of 20.
6) 9 out of 25
7) 34 out of 50
8) 14 out of 20
9) 17 out of 25
10) 21 out of 30
11) Find 25% of 40
12) Find 25% of 80
13) Find 75% of 20
14) Find 75% of 200
15) Find 5% of 40
16) Find 5% of 60
17) Find 15% of 60
18) Find 35% of 65
19) Find 65% of 85
20) Find 75% of 160

Answers

1)	6	6)	36%	11)	10	16)	3
2)	9	7)	68%	12)	20	17)	9
3)	6	8)	70%	13)	15	18)	22.75
4)	30	9)	68%	14)	150	19)	55.25
5)	16	10)	70%	15)	2	20)	120

HCF / LCM using Venn Diagrams

Venn Diagrams – Set Notation

Sets are collections of things

We call the things elements

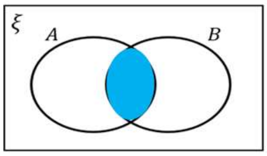
Elements in a set are shown in { }

ξ is the universal set
– this is the group that the elements of a set are selected from

This is called the intersection of A and B

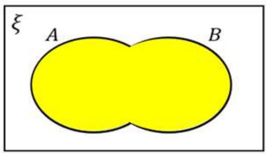
We write this as $A \cap B$

It is a subset of A and of B



This is called the union of A and B

We write this as $A \cup B$

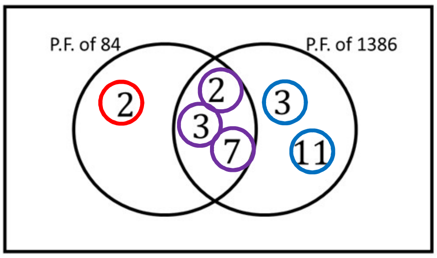


Filling in a Venn Diagram using prime factors

Add common factors to the intersection

$84 = 2 \times 2 \times 3 \times 7$

$1386 = 2 \times 3 \times 3 \times 7 \times 11$



Add the remaining prime factors to complete the sets

Common factors

2

3

7

$2 \times 3 = 6$

$2 \times 7 = 14$

$3 \times 7 = 21$

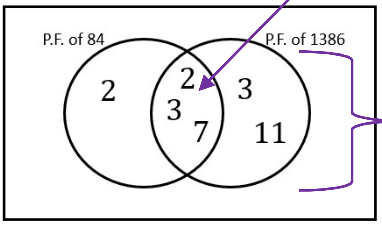
$2 \times 3 \times 7 = 42$

Finding highest common factor (HCF) and lowest common multiple (LCM)

Using a Venn Diagram

$84 = 2 \times 2 \times 3 \times 7$

$1386 = 2 \times 3 \times 3 \times 7 \times 11$



All common factors can be found by finding the products of the prime factors that are in the intersection.
The HCF is the product of all the primes in the intersection.

The LCM is the product of the primes in the union of both sets

Using prime factors

$84 = 2 \times 2 \times 3 \times 7$

$1386 = 2 \times 3 \times 3 \times 7 \times 11$

$\text{HCF}\,(84, 1386) = 2 \times 3 \times 7$

$84 = 2 \times 2 \times 3 \times 7$

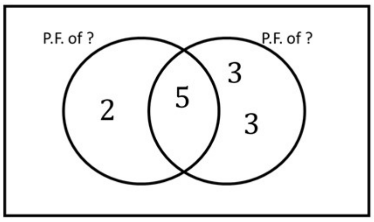
$1386 = 2 \times 3 \times 3 \times 7 \times 11$

$\text{LCM}\,(84, 1386) = 2 \times 3 \times 7 \times 2 \times 3 \times 11$

Problem solving with HCF and LCM

The HCF is 5 and the LCM is 90. What could the two numbers be?

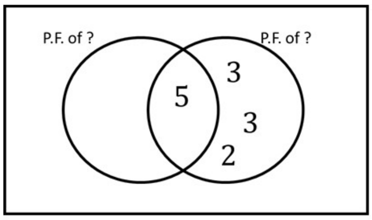
$90 = 2 \times 3 \times 3 \times 5$



$2 \times 5 = 10$ $5 \times 3 \times 3 = 45$

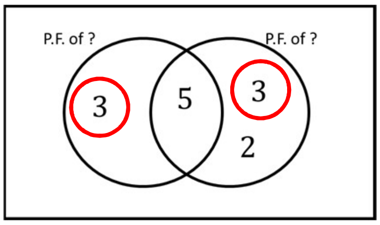
or

.. 5 is in the intersection, complete the diagram using other factors of 90



5 $5 \times 2 \times 3 \times 3 = 90$

.. but not this solution because 3 would be in the intersection

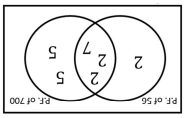


Your turn to practice:

1. Draw a Venn diagram to show the prime factors of 56 and 700
2. **Use your Venn diagram** to find the HCF of 56 and 700
3. **Use your Venn diagram** to find the LCM of 56 and 700
4. By **finding the prime factors**, find the HCF of 56 and 84
5. By **finding the prime factors**, find the LCM of 56 and 84
6. The HCF of two numbers is 10, the LCM is 210. What could the two numbers be?

Answers

1.



2. HCF =28
3. LCM = 1400
4. HCF = 28
5. LCM = 168
6. 70 and 30

Linear Inequalities

Keywords and Phrases:

Inequalities – **Inequality** tells us about the **relative size** of two values.

Critical values – The value in the inequality, e.g

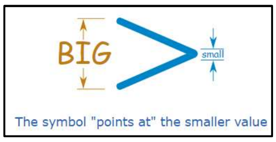
$x < 3$ the critical value will be 3.

$-2 < x \leq 6$ the critical values will be -2 and 6.

Integer – A number with no fractional part (no decimals)

e.g. -2, -3, -1, 0, 1, 2, 3, 4, 5, are all integers.

Symbol	Words
>	greater than
<	less than
≥	greater than or equal to
≤	less than or equal to

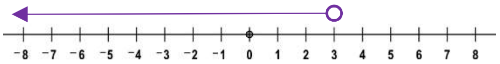


The symbol "points at" the smaller value

Representing inequalities on a number line

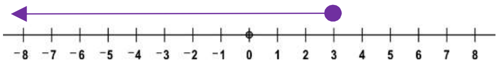
$x < 3$ "x is less than three"

What integer values could x take? $2, 1, 0, -1$ ...



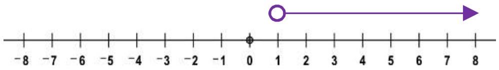
$x \leq 3$ "x is less than or equal to three"

What integer values could x take? $3, 2, 1, 0, -1$ ...



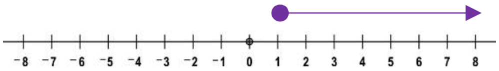
$x > 1$ "x is greater than one"

What integer values could x take? $2, 3, 4$ ...



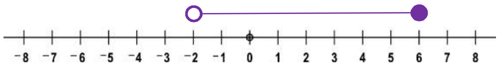
$x \geq 1$ "x is greater than or equal to one"

What integer values could x take? $1, 2, 3,$ ...



$-2 < x \leq 6$ "x is greater than negative two and less than or equal to six"

What integer values could x take? $-1, 0, 1, 2, 3, 4, 5, 6$



Key Points:

●	≤ or ≥	equal to
○	< or >	NOT equal to

Solving inequalities

Use formal working to solve inequalities, we use the same steps because they all of the same critical values

Example 1

Solve $3x + 1 = 10$

$$3x+\overset{0}{1} = 10$$
$$3x + \cancel{1} = 9 + \cancel{1}$$
$$-1 \quad \Big| \quad -1$$
$$3x = 9$$
$$\times\frac{1}{3} \quad \Big| \quad \times\frac{1}{3}$$
$$x = 3$$

Solve $3x + 1 < 10$

$$3x+\overset{0}{1} < 10$$
$$3x + \cancel{1} < 9 + \cancel{1}$$
$$-1 \quad \Big| \quad -1$$
$$3x < 9$$
$$\times\frac{1}{3} \quad \Big| \quad \times\frac{1}{3}$$
$$x < 3$$

Solve $3x + 1 \geq 10$

$$3x+\overset{0}{1} \geq 10$$
$$3x + \cancel{1} \geq 9 + \cancel{1}$$
$$-1 \quad \Big| \quad -1$$
$$3x \geq 9$$
$$\times\frac{1}{3} \quad \Big| \quad \times\frac{1}{3}$$
$$x \geq 3$$

Example 2

Solve $6x - 4 = 2x + 1$

$$\overset{0}{\cancel{2x}} + 4x - 4 = \overset{0}{\cancel{2x}} + 1$$
$$-2x \quad \Big| \quad -2x$$
$$4x - \overset{0}{\cancel{4}} = \quad +1$$
$$+4 \quad \Big| \quad +4$$
$$4x = 5$$
$$\times\frac{1}{4} \quad \Big| \quad \times\frac{1}{4}$$
$$x = \frac{5}{4}$$

Solve $6x - 4 < 2x + 1$

$$\overset{0}{\cancel{2x}} + 4x - 4 < \overset{0}{\cancel{2x}} + 1$$
$$-2x \quad \Big| \quad -2x$$
$$4x - \overset{0}{\cancel{4}} < \quad +1$$
$$+4 \quad \Big| \quad +4$$
$$4x < 5$$
$$\times\frac{1}{4} \quad \Big| \quad \times\frac{1}{4}$$
$$x < \frac{5}{4}$$

Solve $6x - 4 \geq 2x + 1$

$$\overset{0}{\cancel{2x}} + 4x - 4 \geq \overset{0}{\cancel{2x}} + 1$$
$$-2x \quad \Big| \quad -2x$$
$$4x - \overset{0}{\cancel{4}} \geq \quad +1$$
$$+4 \quad \Big| \quad +4$$
$$4x \geq 5$$
$$\times\frac{1}{4} \quad \Big| \quad \times\frac{1}{4}$$
$$x \geq \frac{5}{4}$$

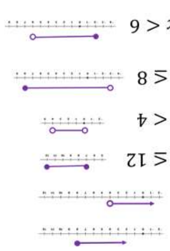
Your turn to practice

In your books, draw a number line from -4 to 12 for each question.

Represent the following inequalities on each number line

1) $x \leq 8$
2) $x < 4$
3) $7 \leq x \leq 12$
4) $0 < x < 4$
5) $-3 < x \leq 8$
6) $-2 \leq x < 6$
7) $x + 7 = 10$
8) $x - 5 < 11$
9) $7 \leq x + 4$
10) $7 + x = 7$
11) $2x \geq 20$
12) $3x + 1 = 10$
13) $4x + 2 \geq 22$
14) $5x + 3 < 18$
15) $2n - 3 = 1$
16) $4a + 3 \leq 3$
17) $2x + 4 = x - 3$
18) $x - 3 \leq 3x + 7$
19) $a - 3 > 3a - 7$

Answers



$x \leq 8$

$x < 4$

$7 \leq x \leq 12$

$0 < x < 4$

$-3 < x \leq 8$

$-2 \leq x < 6$

7) $x = 3$
8) $x < 16$
9) $x \geq 3$
10) $x = 0$
11) $x \geq 10$
12) $x = 3$
13) $x \geq 5$
14) $x < 3$
15) $n = 2$
16) $a \leq 0$
17) $x = -7$
18) $x \geq -5$
19) $a < 2$

Division

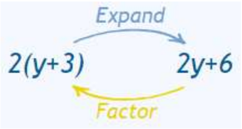
Keywords and Phrases:

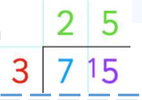
Inverse - Inverse means the opposite in effect. The reverse of.

Factorise – the process of finding factors.

Finding what to multiply together to get an expression.

Division – There are lots of different ways of dividing, in primary school many children have been taught a “chunking” method and then the “bus stop” method.

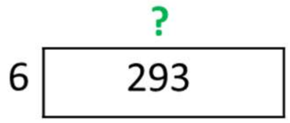


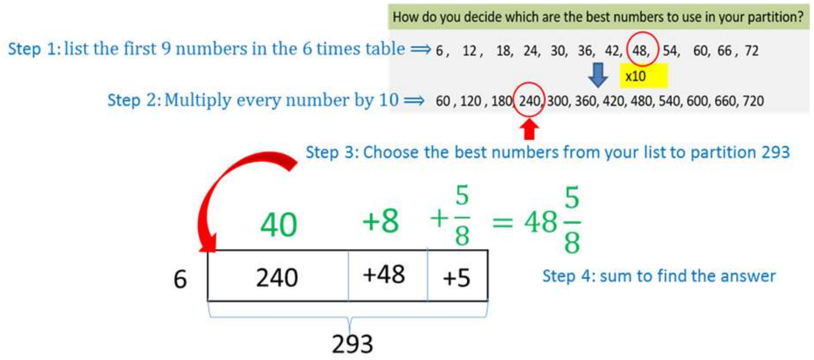


Inverse of area model for division of large numbers

This model is very similar to the chunking methods used in primary, but represented using an area model.

$$293 \div 6 =$$




How do you decide which are the best numbers to use in your partition?

Step 1: list the first 9 numbers in the 6 times table ⟹ 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72

x10

Step 2: Multiply every number by 10 ⟹ 60, 120, 180, 240, 300, 360, 420, 480, 540, 600, 660, 720

Step 3: Choose the best numbers from your list to partition 293

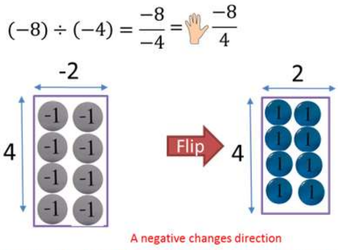
$$40 \quad +8 \quad +\frac{5}{8} = 48\frac{5}{8}$$

Step 4: sum to find the answer

Dividing negative numbers

There are two representations for this:

Using counters to create an array:



Using fractions as division:

$$(-8) \div (-4) = \frac{-8}{-4}$$

$$= -\frac{-8}{4}$$

$$= -\frac{-2 \times 4}{4}$$

$$= -(-2) \times \frac{4}{4}$$

$$= 2 \times 1$$

$$= 2$$

Factorising

Factorise $3x + 6$

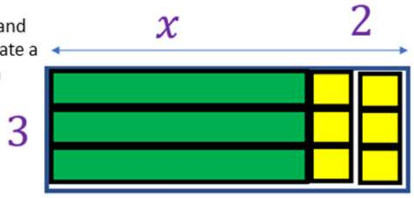
Step 1 - What is the HCF of $3x$ and 6?

Factor pairs of $3x$	Factor pairs of 6
$1 \times 3x$	1×6
$3 \times x$	3×2

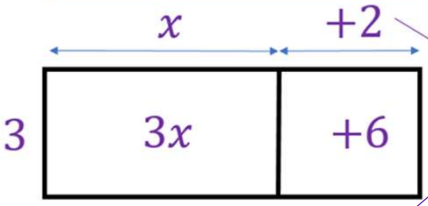
HCF of $3x$ and 6 = 3

$\text{HCF}(3x, 6) = 3$

Using 3 x counters and 6 unit counters, create a rectangle that has a width of 3



You could use an area model like this to show the amount of algebra tiles.



The side lengths are the answer.

$$3x + 6 \equiv 3(x + 2)$$

Your turn to practice

Calculate the following using area model for division, leaving your answer as a mixed number fraction

1) $228 \div 3$
2) $528 \div 3$
3) $528 \div 4$
4) $524 \div 3$
5) $169 \div 8$

6) $(+6) \div (-3) =$
7) $(-12) \div (-6) =$
8) $(-10) \div (-5) =$
9) $(+15) \div (-5) =$
10) $(-16) \div (-4) =$

Factorise:

11) $2x + 4$
12) $10x + 5$
13) $6x - 18$
14) $18y - 27x$
15) $60x - 36xy$

Answers

1)	76
2)	176
3)	132
4)	$174\frac{2}{3}$
5)	$21\frac{1}{8}$
6)	-2
7)	2
8)	2
9)	-3
10)	4
11)	$2(x + 2)$
12)	$5(2x + 1)$
13)	$6(x - 3)$
14)	$9(2y - 3x)$
15)	$12x(5 - 3y)$

Standard form

Keywords and Phrases:

Standard form - A general term meaning "written down in the way most commonly accepted"
This common way depends upon the subject and country, in the UK we use "Scientific Notation"
Standard form can also be referred to as standard index form.

Digits Power of 10
$5326.6 = 5.3266 \times 10^3$
A Number In Scientific Notation

Standard form:

To be in standard form a number must be written as:

$$a \times 10^m \quad \text{where } 1 \le a < 10 \text{ and } m \text{ is an integer}$$

Using a number line can help, as per below:

E.g:
Convert 5 000 000 into standard form

$$\boxed{= 5 \times 10^6}$$

Billion			Million			Thousands			Unit			Decimals		
Hundred billions	Ten Billions	billion	Hundred Millions	Ten Millions	One Million	Hundred Thousands	Ten thousands	Thousand	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
10^{11}	10^{10}	10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
100 000 000 000	10 000 000 000	1 000 000 000	100 000 000	10 000 000	1 000 000	100 000	10 000	1 000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
					5	0	0	0	0	0	0			

E.g: 2 750 000 $= 2.75 \times 10^6$ For these types of numbers use the most significant number

Standard form is also used for really small numbers:

E.g: 0.004 $= 4 \times 10^{-3}$ 0.00000012 $= 1.2 \times 10^{-7}$

You will also need to know how to convert numbers from standard form into ordinary numbers

Adjusting into standard form:

Sometimes a number looks like it is in standard form, but it is not. You need to adjust it into standard form:

E.g: 12×10^2 This number is not in standard form

Adjust 12 to be in standard form:
$12 = 1.2 \times 10^1$

$1.2 \times 10^1 \times 10^2$

Standard form
$\boxed{= 1.2 \times 10^3}$

E.g: 0.00012×10^5 This number is not in standard form

Adjust to standard form:
$0.00012 = 1.2 \times 10^{-4}$

$1.2 \times 10^{-4} \times 10^5$

Standard form
$\boxed{= 1.2 \times 10^1}$

Ordering in standard form:

Example: Put these numbers in order of size, starting with the smallest?

12.2×10^2 1.22×10^5 122×10^{-3} 0.00122×10^7

There are a few ways of ordering with standard form.

Order by converting into ordinary numbers:

$12.2 \times 10^2 = 1\,220$ 2nd
$1.22 \times 10^5 = 122\,000$ 4th
$122 \times 10^{-3} = 0.122$ 1st
$0.00122 \times 10^7 = 12\,200$ 3rd

Or convert them all into standard form:

$12.2 \times 10^2 = 1.2 \times 10^3$ 2nd
1.22×10^5 4th
$122 \times 10^{-3} = 1.22 \times 10^{-1}$ 1st
$0.00122 \times 10^7 = 1.22 \times 10^4$ 3rd

Smallest 122×10^{-3} 12.2×10^2 0.00122×10^7 1.22×10^5 Largest

Your turn to practice

Convert these numbers into standard form:

1) 80 000
2) 9 000
3) 410 000
4) 4 600
5) 450
6) 0.04
7) 0.000 000 005
8) 0.0234
9) 0.000 0023
10) 0.0067

Convert these numbers into ordinary numbers:

11) 5×10^7
12) 9×10^8
13) 3.7×10^9
14) 2.8×10^1
15) 9.9×10^5
16) 3.2×10^{-5}
17) 2.9×10^{-2}
18) 3.167×10^{-1}
19) 1.115×10^{-4}
20) 1.412×10^{-3}

Order these numbers from smallest to largest:

21) 9×10^5 9×10^3 9×10^2 9×10^7
22) 3×10^5 3×10^{-3} 3×10^2 3×10^{-7}
23) 2×10^3 5×10^3 9.2×10^3 6.3×10^3
24) 4×10^7 7×10^4 3×10^4 5×10^7
25) 83000 8×10^4 8.3×10^3 8000

Answers

1) 8×10^4
2) 9×10^3
3) 4.1×10^5
4) 4.6×10^3
5) 4.5×10^2
6) 4×10^{-2}
7) 5×10^{-9}
8) 2.34×10^{-2}
9) 2.3×10^{-6}
10) 6.7×10^{-3}
11) 50000000
12) 900000000
13) 3700000000
14) 28
15) 99000
16) 0.000032
17) 0.029
18) 0.3167
19) 0.0001115
20) 0.001412

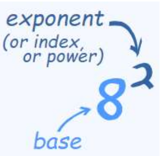
21) 9×10^2 9×10^3 9×10^5 9×10^7
22) 3×10^{-7} 3×10^{-3} 3×10^2 3×10^5
23) 2×10^3 5×10^3 6.3×10^3 9.2×10^3
24) 3×10^4 7×10^4 4×10^7 5×10^7
25) 8000 8.3×10^3 8×10^4 83000

Exponents

Keywords and Phrases:

Exponent - Exponents are also called **Powers** or **Indices** or Index. The exponent of a number says **how many times** to use the number in a **multiplication.**

Base number – In the example of 8^2, 8 is the base number.



In this example:
$\mathbf{8^2 = 8 \times 8 \ = 64}$

Multiplication rule of Exponents:

$$2^3 \times 2^4 = \overbrace{\underbrace{(2\times2\times2)}_{3} \times \underbrace{(2\times2\times2\times2)}_{4}}^{7}$$

$$= 2^7$$

Could we work out the answer without writing it out the calculation in full?

$$2^3 \times 2^4 = 2^{3+4}$$
$$= 2^7$$

$$a^m \times a^n = a^{m+n}$$

For this rule to work the base numbers must be the same.

Division rule of Exponents:

$$\frac{2^7}{2^3} = \frac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2}$$
$$= \overbrace{\underbrace{\frac{2}{2}\times\frac{2}{2}\times\frac{2}{2}}_{3} \times \underbrace{2\times2\times2\times2}_{4}}^{7}$$

Could we work out the answer without writing it out the calculation in full?

$$= 1\times1\times1\times2^4$$
$$= 2^4$$

$$2^7 \div 2^3 = 2^{7-3}$$
$$= 2^4$$

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

For this rule to work the base numbers must be the same.

Brackets rule of Exponents:

$$(2^4)^3 = 2^4 \times 2^4 \times 2^4$$

$$= \overbrace{\underbrace{(2\times2\times2\times2)}_{4} \times \underbrace{(2\times2\times2\times2)}_{4} \times \underbrace{(2\times2\times2\times2)}_{4}}^{3\times4=12}$$

$$= 2^{12}$$

Could we work out the answer without writing it out the calculation in full?

$$(2^4)^3 = 2^{4\times3}$$
$$= 2^{12}$$

$$(a^m)^n = (a^n)^m = a^{nm}$$

For this rule to work the base numbers must be the same.

Zero Exponent:

Anything to the power of zero = 1

$$a^0 = 1 \qquad a^1 = a$$

Negative Exponents:

Calculate the positive exponent and the take the reciprocal of it.

E.g: Work out the value of 2^{-1} $\quad 2^{-1} = \frac{1}{2}$

E.g: Work out the value of $\left(\frac{2}{3}\right)^{-2}$ $\quad \left(\frac{2}{3}\right)^{-2} = \frac{3^2}{2^2} \quad = \frac{9}{4}$

$$a^{-m} = \frac{1}{a^m}$$

a^{-m} is the reciprocal of a^m

Fractional Exponents:

$\sqrt{a} = a^{\frac{1}{2}}$

Work out the value of $121^{\frac{1}{2}}$

$121^{\frac{1}{2}} = \sqrt{121} = 11$

$\sqrt[3]{a} = a^{\frac{1}{3}}$

Work out the value of $27^{\frac{1}{3}}$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$$\sqrt[m]{a} = a^{\frac{1}{m}}$$

Your turn to practice

1) $2^6 \times 2^8 =$
2) $5^7 \times 5^2 =$
3) $7^3 \times 7 =$
4) $2.1^5 \times 2.1^4 =$
5) $2^9 \div 2^6 =$
6) $\frac{5^7}{5^2} =$
7) $7^{11} \div 7^3 =$
8) $5.2^5 \div 5.2 =$
9) $(2^5)^4 =$
10) $(5^3)^6 =$
11) $(7^3)^7 =$
12) $(5.2^2)^9 =$
13) $2^0 =$
14) $52^0 =$
15) $7.5^0 =$
16) $1620^0 =$
17) $81^{\frac{1}{2}}$
18) $144^{\frac{1}{2}}$
19) $400^{\frac{1}{2}}$
20) $1.21^{\frac{1}{2}}$
21) $8^{\frac{1}{3}}$
22) $64^{\frac{1}{3}}$
23) $125^{\frac{1}{3}}$
24) $0.125^{\frac{1}{3}}$

Answers

1)	2^{14}	13)	1
2)	5^9	14)	1
3)	7^4	15)	1
4)	2.1^9	16)	1
5)	2^3	17)	9
6)	5^5	18)	12
7)	7^8	19)	20
8)	5.2^4	20)	1.1
9)	2^{20}	21)	2
10)	5^{18}	22)	4
11)	7^{21}	23)	5
12)	5.2^{18}	24)	$\frac{1}{2}$

Collecting like terms

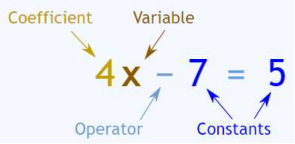
Keywords and Phrases:

Like terms - are terms whose variables (and their exponents such as the 2 in x^2) are the same.

E.g: $5x$ $7x$ $-x$ $12x$ are all like terms because the variables are all x

Coefficients - A number used to multiply a variable.

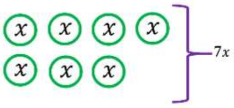
E.g: $6z$ means 6 times z, and "z" is a variable, so 6 is a coefficient.



Collecting Like Terms – Linear expressions

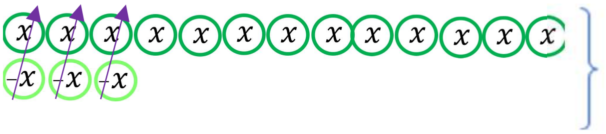
A term is separated by a (+) or (–) sign in an expression. When you collect the terms you are adding them together. Like terms have the same variable such as x or y

$4x + 3x \equiv$

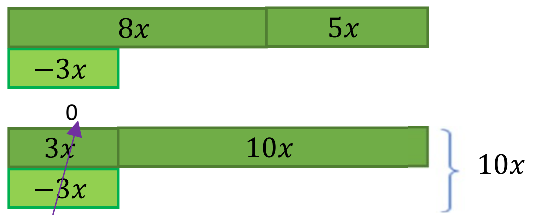


Or using a bar model

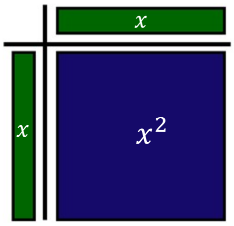
$8x - 3x + 5x \equiv \quad (+8x) + (-3x) + (+5x) \equiv$



$10x$

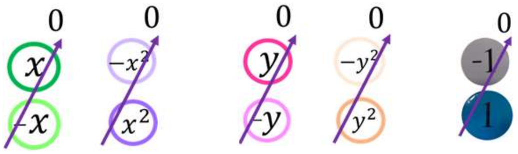


Key points - Quadratics:



$x \times x \quad \equiv x^2$

Zero pairs:



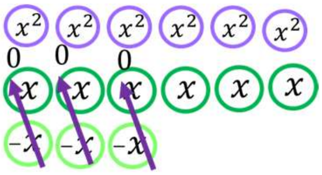
Collect like terms - Quadratic

Example 1: $4x^2 + 6x - 3x + 2x^2 \;=\; (+4x^2) + (+2x^2) + (+6x) + (-3x)$

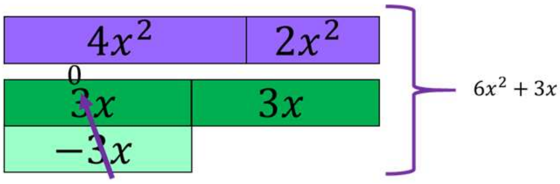
Like terms
$(+4x^2) + (+2x^2)$

Like terms
$(+6x) + (-3x)$

We could use counters to represent this:



We could use bar model to represent this:

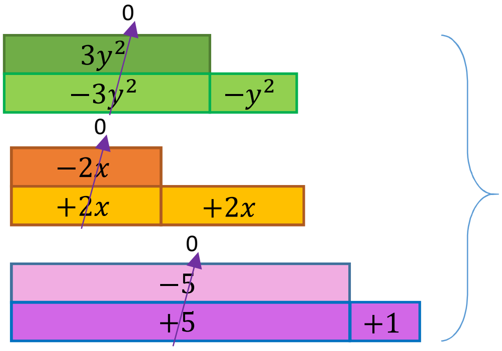


Example 2: $3y^2 - 2x - 5 + 4x - 4y^2 + 6 \;=\; (+3y^2) + (-4y^2) + (-2x) + (+4x) + (-5) + (+6)$

Like terms
$(+3y^2) + (-4y^2)$

Like terms
$(-2x) + (+4x)$

Like terms
$(-5) + (+6)$



$\equiv -y^2 + 2x + 1$

Your turn to practice

1) $x^2 - y^2 + x^2 - y^2$
2) $3x^2 + 2y^2 + 2x^2 + y^2$
3) $3x^2 + 2y^2 - 2x^2 + y^2$
4) $-3x^2 + 2y^2 - 2x^2 - y^2$
5) $2x^2 + x^2 + y^2 + y^2 + 2x^2 + y^2$
6) $3x^2 + 2x^2 + 3x + 4y^2 + y^2 - 4x + 3$
7) $5y^2 + 3x^2 + 2y + 2y^2 + 5y - 3x$
8) $4x^2 + 2x + 2 + 5x^2 - 6y^2 + 11x + 2y^2 - 15$
9) $-7x^2 + 7x + 3x^2 + 7y - 10x + 3y$
10) $2x^2 - 3x + 3x^2 + 2y^2 + 5x + 6y^2 + 2y$

Answers

1) $2x^2 - 2y^2$
2) $5x^2 + 3y^2$
3) $x^2 + 3y^2$
4) $-5x^2 + y^2$
5) $5x^2 + 3y^2$
6) $5x^2 - x + 5y^2 + 3$
7) $7y^2 + 3x^2 + 7y - 3x$
8) $9x^2 - 4y^2 + 13x - 13$
9) $-4x^2 - 3x + 10y$
10) $5x^2 + 8y^2 + 2x + 2y$

BIPS and Substitution

Keywords and Phrases:

BIDMAS or BIPS - BIDMAS gives us a rule to follow for the order of our operations.

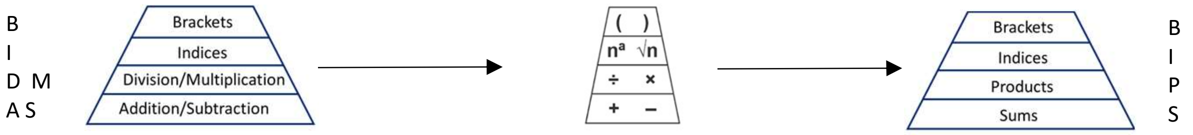
Formula - A formula is a fact or rule that uses mathematical symbols. Has an equals sign and at least two different variables.

Substitution - In Algebra "Substitution" means putting numbers in place of where the letters are

BIPS

Sometimes referred to as BIDMAS.

Since we can always rewrite a division as a multiplication and a subtraction as an addition, we are going to use…



Calculate these by working left to right:

$$2+4-3 \xrightarrow{\text{Alternative calculation}} 2+4+(-3)$$

We have turned this into an addition of a negative and so now we can use the commutative law.

$$2-3+4 \xrightarrow{\text{Alternative calculation}} 2+(-3)+4$$

Calculate these by working left to right:

$$20\times 4\div 2 \xrightarrow{\text{Alternative calculation}} 20\times 4\times\frac{1}{2}$$

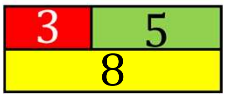
We have turned this into a multiplication by a fraction and so now we can use the commutative law.

$$20\div 2\times 4 \xrightarrow{\text{Alternative calculation}} 20\times\frac{1}{2}\times 4$$

Substituting into a formula

Work out the value of

Example 1: $y=r+l$ Substitute $r=3$ and $l=5$ to evaluate y

	Bar model method	Formal method
Step 1 State the formula		$y=r+l$
Step 2 Substitute in values		$y=3+5$
Step 3 Use BIPS to solve		$y=8$

Example 2: $v=u+at$

Evaluate v when $u=10, a=-4\ t=6$

Step 1
State the formula

$$v=u+at$$

Step 2
Substitute in values

$$v=10+(-4)\times 6$$

$$v=10+(-24)$$

Step 3
Use BIPS to solve

$$v=-14$$

Key misconceptions:

Evaluate $3x^2$ when $x=5$

$$3x^2=3\times 5^2$$
$$=15^2$$
$$=225$$ ✗

$$3x^2=3\times 5^2$$
$$=3\times 25$$
$$=75$$ ✓

Just x is squared

$$3x^2=3\times x^2$$
$$(3x)^2=(3\times x)^2$$

$3\times x$ is squared

Your turn to practice

1) $4+1\times 5$
2) $6\div 3+9$
3) $6\times(5-2)$
4) $36-2+32\div 8$
5) $(16-3)+8\div 2$
6) $(8+2\times 4)^2$
7) $(5-12)^2$
8) 6×3^2
9) $-11-50\div\sqrt{25}\times 2$
10) $(12-2)^4-(47+3^2)$

11) If $a=6$ and $b=2$ find the value of:
- a. $a+b$
- b. $3a+2b$
- c. $5b-a$
- d. $2a^2-6b$
- e. $\frac{1}{3}ab$
- f. $\frac{a^2}{b}$

12) If $p=5$ and $q=-3$ find the value of:
- a. $p+q$
- b. $2p+5q$
- c. $6p-q$
- d. $4p^2-2q$
- e. $\frac{pq}{5}$
- f. $(2p)^2-q$

Answers
1) 9
2) 11
3) 18
4) 38
5) 17
6) 256
7) 49
8) 54
9) −31
10) 9944
11) a) 8 b) 22 c) 4
d) 60 e) 4 f) 18
12) a) 2 b) -5 c) 33
d) 106 e) -3 f) 103

Changing the subject of a formula

Keywords and Phrases:

Formula - A formula is a fact or rule that uses mathematical symbols. Has an equals sign and at least two different variables.

Subject - The "subject" of a formula is the single variable (usually on the left of the "=") to which everything else is equal.

$$s = ut + \frac{1}{2}at^2$$ s is the subject of this formula

Changing the subject of a formula – using bar models:

$$y = r + l$$

y is the subject of the formula

Make r the subject of the formula:

$$y = r + \cancel{l}$$
$$-l \qquad \cancel{+l}$$
$$y - l = r$$

r is the subject of the formula

$$r = y - l$$

$$b = 2r + l$$

b is the subject of the formula

Make r the subject of the formula:

$$b = 2r + \cancel{l}$$
$$-l \qquad \cancel{+l}$$
$$b - l = 2r$$
$$\div 2 \qquad \div 2$$
$$\frac{b-l}{2} = r$$

r is the subject of the formula

Or can be written like this:

$$r = \frac{b-l}{2}$$

$$r = \frac{1}{2}(b-l)$$

Changing the subject of a formula – using formal methods only:

Make x the subject of these formulas:

1)

$$y = 3x - \cancel{7}$$
$$+7 \qquad \cancel{+7}$$
$$y + 7 = 3x$$
$$\div 3 \qquad \div 3$$
$$\frac{y+7}{3} = x$$

2)

$$y = \frac{x}{2} + \cancel{3}$$
$$-3 \qquad \cancel{-3}$$
$$y - 3 = \frac{x}{2}$$
$$\times 2 \qquad \times 2$$
$$2(y-3) = x$$
$$\Updownarrow$$
$$2y - 6 = x$$

3)

$$y = \frac{x+5}{3}$$
$$\times 3 \qquad \times 3$$
$$3y = x + \cancel{5}$$
$$-5 \qquad \cancel{-5}$$
$$3y - 5 = x$$

Your turn to practice

Make y the subject of these formulas:

1) $y + w = c$
2) $y - 2g = n$
3) $3y = c$
4) $ay = w$
5) $\frac{y}{c} = w$
6) $c = y - k$

Make x the subject of these formulas:

7) $4x + c = w$
8) $dx - t = 8$
9) $2x + 2y = P$
10) $y = xz + s$
11) $3y = 4x + 1$
12) $\frac{x+t}{m} = 2c$

Answers

1) $y = c - w$
2) $y = n + 2g$
3) $y = \frac{c}{3}$
4) $y = \frac{w}{a}$
5) $y = wc$
6) $c + k = y$
7) $x = \frac{w-c}{4}$
8) $x = \frac{8+t}{d}$
9) $x = \frac{P-2y}{2}$
10) $\frac{y-s}{z} = x$
11) $\frac{3y-1}{4} = x$
12) $x = 2cm - t$

Single bracket - Quadratic

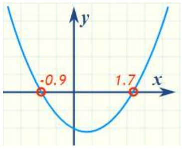
Keywords and Phrases:

Quadratic - The name **Quadratic** comes from "quad" meaning square, because the variable is squared (like **x^2**).

The standard form to see a quadratic is: $ax^2 + bx + c$

Where a, b and c are known values, $a \neq 0$.

This is a quadratic curve



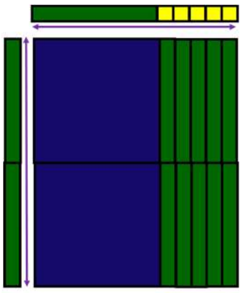
This makes it quadratic

$5x^{②} + 3x + 3 = 0$

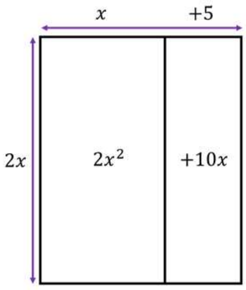
Expanding a single term over a bracket (Quadratic):

Expand $2x(x + 5)$

Using algebra tiles

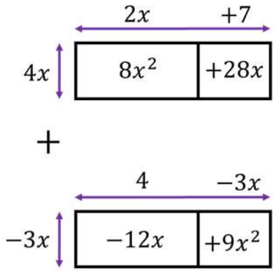


Using area model



$2x(x + 5) \equiv 2x^2 + 10x$

Expand and simplify $4(2x + 7) - 3(4 - 3x)$



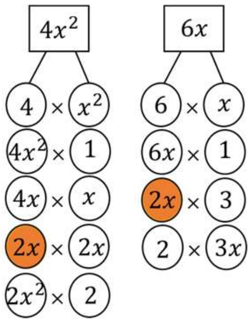
$$\begin{array}{r} 8x^2 + 28x \\ + \quad 9x^2 - 12x \\ \hline 17x^2 + 16x \end{array}$$

$4x(2x + 7) - 3x(4 - 3x) \equiv 17x^2 + 16x$

Factorise a single term over a bracket (Quadratic):

Factorise fully $4x^2 + 6x$

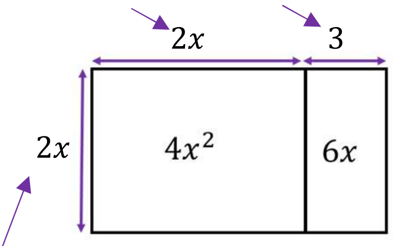
Find the highest common factor of $4x^2$ and $6x$



HCF $(4x^2, 6x) = 2x$

Area Model

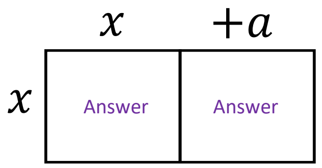
If the area is $4x^2 + 6x$, what can the side lengths be?



$4x^2 + 6x = 2x(2x + 3)$

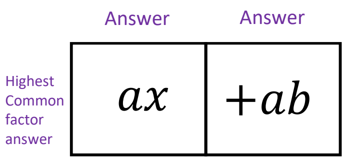
Factorising or Expanding?

Expand $x(x + a)$ Take out of bracket



Write answer out of area model $= x^2 + ax$

Factorise $ax + ab$ Put in bracket



Write answer out of area model $= a(x + b)$

Your turn to practice

Expand using area model

1) $2x(x + 4)$

2) $3x(x - 2)$

3) $2x(3x + 1)$

4) $5(2x - 3)$

5) $-2x(x + 7)$

6) $-3x(3x - 2)$

7) $\frac{1}{2}x(x + 6)$

8) $x(3x + 2y)$

9) $4x(x + 5) + 2x(x + 4)$

10) $2x(x - 4) + 3x(x + 6)$

11) $5x(2x + 1) + 4x(4x + 3)$

12) $3x(3x - 7) + x(2x + 9)$

13) $2x(3x + 1) - x(6x - 3)$

14) $7x(3x + 5) - 5x(3x - 4)$

15) $2x(9x - 2) - 6x(5x - 9)$

16) $4x(5x + 3) - 3x(2 - 3x)$

Factorise using area model

17) $x^2 + 4x$

18) $2x^2 + 4x$

19) $4x^2 + 4x$

20) $-4x^2 + 4x$

21) $-4x^2 - 4x$

22) $-4x^2y - 4xy$

23) $3x^2 + 6x$

24) $6x^2 + 9x$

Answers

1) $2x^2 + 8x$

2) $3x^2 - 6x$

3) $6x^2 + 2x$

4) $10x - 15$

5) $-2x^2 - 14x$

6) $-9x^2 + 6x$

7) $\frac{1}{2}x^2 + 3x$

8) $3x^2 + 2xy$

9) $6x^2 + 28x$

10) $5x^2 + 10x$

11) $26x^2 + 17x$

12) $11x^2 - 12x$

13) $5x$

14) $6x^2 + 55x$

15) $-12x^2 + 50x$

16) $29x^2 + 6x$

17) $x(x + 4)$

18) $2x(x + 2)$

19) $4x(x + 1)$

20) $-4x(x - 1)$

21) $-4x(x + 1)$

22) $-4xy(x + 1)$

23) $3x(x + 2)$

24) $3x(2x + 3)$

Expand a Double bracket

Keywords and Phrases:

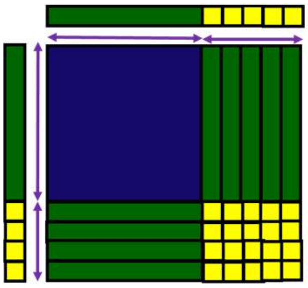
Expand – Means to multiply out the brackets, using algebra tiles or an area model.

Area model - A model used for multiplication, each rectangle represents an area, the side lengths are the question. The area is the answer.

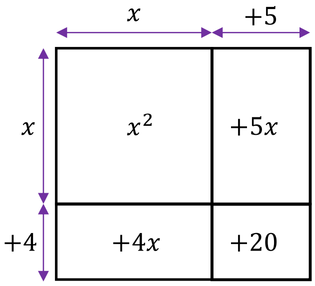
Expanding double brackets - both positive:

Expand $(x+4)(x+5)$

Using algebra tiles



Using area model



Collect like terms $4x+5x=9x$

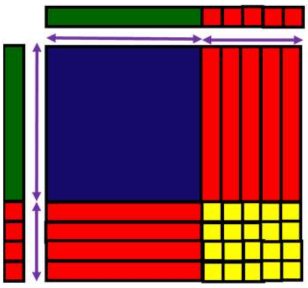
$$\equiv x^2+4x+5x+20$$

$$\equiv x^2+9x+20$$

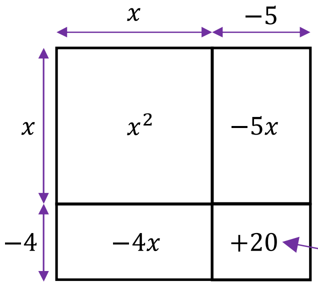
Expanding double brackets - both negative:

Expand $(x-4)(x-5)$

Using algebra tiles



Using area model



Collect like terms $(-4x)+(-5x)=(-9x)$

$$\equiv x^2-4x-5x+20$$

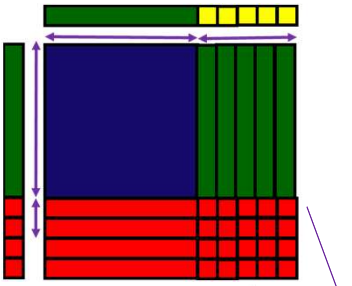
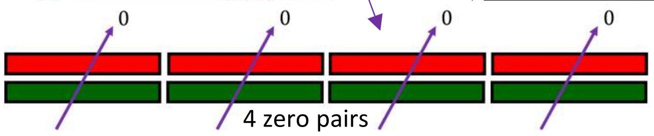
$$\equiv x^2-9x+20$$

Multiplying by a negative changes the direction. $(-5)\times(-4)=+20$

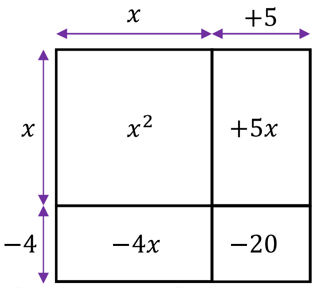
Expanding double brackets - one positive one negative:

Expand $(x-4)(x+5)$

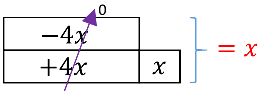
Using algebra tiles

Using area model



Collect like terms Using bar model and zero pairs



$$\equiv x^2-4x+5x-20$$

$$\equiv x^2+x-20$$

Your turn to practice

Expand the following using the area model, and collect like terms:

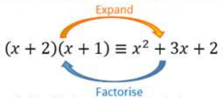
1) $(x+2)(x+4)$	*6)* $(x-2)(x-4)$	*11)* $(x-2)(x+4)$
2) $(x+3)(x+5)$	*7)* $(x-3)(x-5)$	*12)* $(x+3)(x-5)$
3) $(x+6)(x+1)$	*8)* $(x-6)(x-1)$	*13)* $(x-6)(x+1)$
4) $(4+x)(x+4)$	*9)* $(4-x)(x-4)$	*14)* $(4+x)(x-4)$
5) $(x+\frac{1}{2})(x+\frac{2}{3})$	*10)* $(x-\frac{1}{2})(x-\frac{2}{3})$	*15)* $(x-\frac{1}{2})(x+\frac{2}{3})$

Answers

1) x^2+6x+8	*9)* $-x^2+8x-16$
2) $x^2+8x+15$	*10)* $x^2-\frac{7}{6}x+\frac{1}{3}$
3) x^2+7x+6	*11)* x^2+2x-8
4) $x^2+8x+16$	*12)* $x^2-2x-15$
5) $x^2+\frac{7}{6}x+\frac{1}{3}$	*13)* x^2-5x-6
6) x^2-6x+8	*14)* x^2-16
7) $x^2-8x+15$	*15)* $x^2+\frac{1}{6}x-\frac{1}{3}$
8) x^2-7x+6	

Factorise a Quadratic

Keywords and Phrases:



$(x+2)(x+1) \equiv x^2+3x+2$

Factorise – Factorising is the process of **finding the factors.**
Factorising is the opposite to expanding.

Coefficient - A number used to multiply a variable. E.g: $6z$ means 6 times z, and "z" is a variable, so 6 is a coefficient.

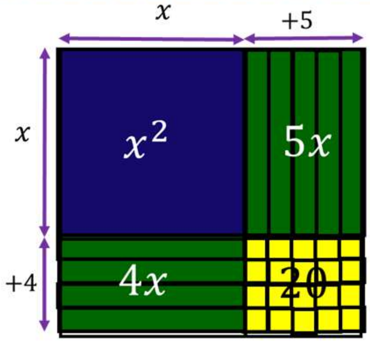
Factorise double brackets – all terms positive:

Factorise $x^2+9x+20$

Multiply to make 20

20
1×20
2×10
4×5

Which two factors sum to give $9x$



$x^2+9x+20 \equiv \mathbf{(x+4)(x+5)}$

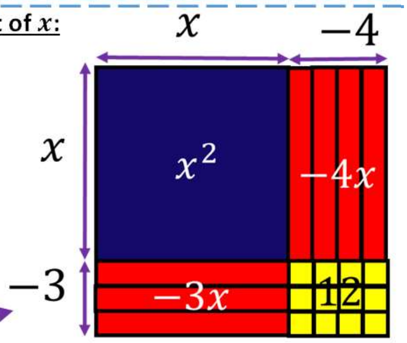
Factorise double brackets – Negative coefficient of x:

Factorise $x^2-7x+12$

Multiply to make 12

12
1×12
-1×-12
2×6
-2×-6
3×4
-3×-4

Which two factors sum to give $-7x$



$x^2-7x+12 \equiv \mathbf{(x-3)(x-4)}$

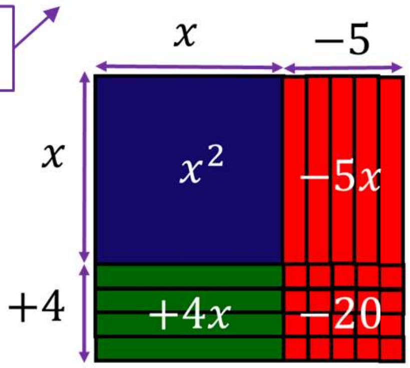
Factorise double brackets – Negative constant:

Factorise x^2-x-20

Multiply to make -20

-20
-1×20
1×-20
-2×10
2×-10
-4×5
4×-5

Which two factors sum to give $-x$



$\mathbf{(x+4)(x-5)}$
$x^2-x-20 \equiv \mathbf{(x-5)(x+4)}$

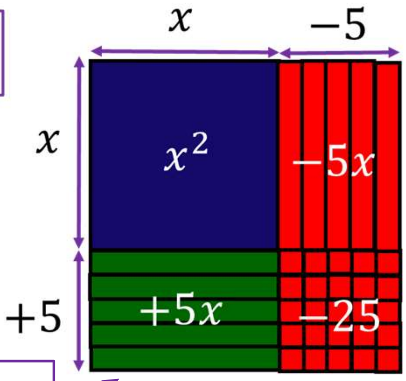
Factorise double brackets – Difference of two squares:

Factorise x^2-25

Multiply to make -25

-25
-1×25
1×-25
-5×5

Which two factors sum to give 0



$\mathbf{(x+5)(x-5)}$
$x^2-25 \equiv \mathbf{(x-5)(x+5)}$

Your turn to practice

1) x^2+4x+3
2) x^2+6x+9
3) x^2+6x+5
4) x^2+7x+6
5) $x^2+8x+16$
6) x^2-4x+3
7) x^2-9x+8
8) x^2-4x+4
9) $x^2-9x+14$
10) $x^2-12x+20$
11) $x^2-7x-18$
12) $x^2-3x-18$
13) $x^2+17x-18$
14) x^2-x-30
15) $x^2-7x-30$

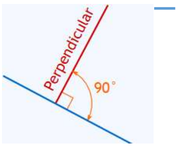
Answers

1) $(x+1)(x+3)$
2) $(x+3)(x+3)$
3) $(x+1)(x+5)$
4) $(x+1)(x+6)$
5) $(x+4)(x+4)$
6) $(x-1)(x-3)$
7) $(x-1)(x-8)$
8) $(x-2)(x-2)$
9) $(x-2)(x-7)$
10) $(x-2)(x-10)$
11) $(x-9)(x+2)$
12) $(x-6)(x+3)$
13) $(x+18)(x-1)$
14) $(x-6)(x+5)$
15) $(x-10)(x+3)$

Area

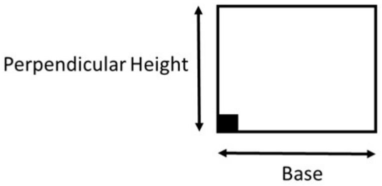
Keywords and Phrases:

Perpendicular – At right angles (90°) to



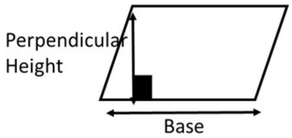
Key Formula:

Area of a Rectangle = Base × Perpendicular Height



$$A = b \times h$$

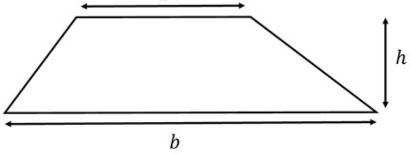
Area of a Parallelogram = Base × Perpendicular Height



$$A = b \times h$$

A rhombus is a parallelogram with four equal sides. This formula also applies to a rhombus.

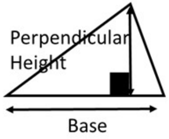
Area of a Trapezium = Half the sum of the parallel sides × Perpendicular Height



$$A = \frac{(a+b)}{2} \times h$$

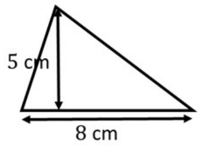
$$A = \frac{1}{2}(a+b) \times h$$

Area of a Triangle = $\frac{1}{2}$ × Base × Perpendicular Height



$$A = \frac{1}{2} \times b \times h$$

Example: Find the area of this triangle.



State the formula first!

$A = \frac{1}{2} \times b \times h$

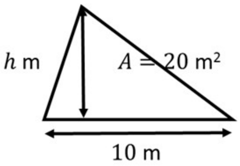
$= \frac{1}{2} \times 8 \times 5$

$= 4 \times 5$

$= 20$ cm²

Always give the units!

Example: Find the perpendicular height of this triangle.



$A = \frac{1}{2} \times b \times h$

$20 = \frac{1}{2} \times 10 \times h$

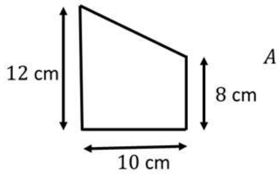
$20 = 5 \times h$

÷ 5 | ÷ 5

$4 = h$

$h = 4$ m

Example: Find the area of this trapezium



State the formula first!

$A = \frac{(a+b)}{2} \times h$

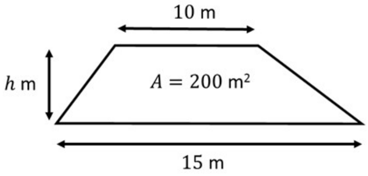
$= \frac{(12+8)}{2} \times 10$

$= 10 \times 10$

$= 100$ cm²

Always give the units!

Example: Find the perpendicular height of this trapezium.



$A = \frac{(a+b)}{2} \times h$

$200 = \frac{15+10}{2} \times h$

× 2 | × 2

$400 = 25 \times h$

÷ 25 | ÷ 25

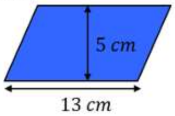
$16 = h$

$h = 16$ m

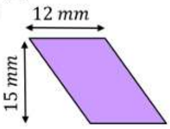
Your turn to practice

1) Calculate the area of these shapes:

(a)

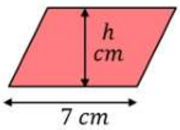


(b)

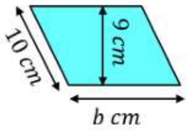


2) Given the area, calculate the missing length:

(a) $Area = 42\ cm^2$

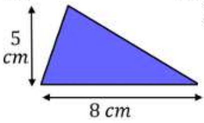


(b) $Area = 67.5\ cm^2$

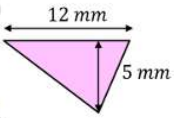


3) Calculate the area of these shapes:

(a)

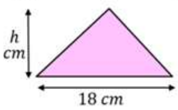


(b)

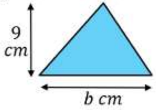


4) Given the area, calculate the missing length:

(a) $Area = 72\ cm^2$

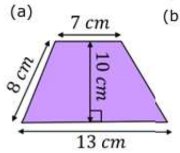


(b) $Area = 22.5\ mm^2$

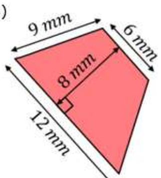


5) Calculate the area of these shapes:

(a)

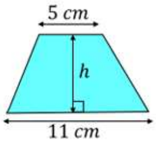


(b)

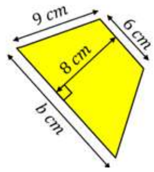


6) Given the area, calculate the missing length:

(a) $Area = 48\ cm^2$



(b) $Area = 72\ cm^2$



Answers

1) a) 65cm²
b) 180mm²

2) a) h = 6cm
b) b = 7.5cm

3) a) 20cm²
b) 30mm²

4) a) h = 8cm
b) b = 5cm

5) a) 100cm²
b) 72mm²

6) a) h = 6cm
b) 12cm

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